Semantics and Necessary Truth

AN INQUIRY INTO THE FOUNDATIONS

OF ANALYTIC PHILOSOPHY

by Arthur Pap

ASSOCIATE PROFESSOR OF PHILOSOPHY, YALE UNIVERSITY

NEW HAVEN: YALE UNIVERSITY PRESS, 1958

Set in Baskerville type and
printed in the United States of America by
J. H. Furst Co., Baltimore, Md.

Library of Congress catalog card number: 58-6543

a priori
|
necessary

empirical
|
contingent

125

PREFACE

THE DISTINCTIONS between a priori knowledge and empirical knowledge ("necessary truth" and "contingent truth") and between analytic and synthetic propositions lie at the very heart of modern epistemology. One of the most important and apparently interminable controversies in contemporary, semantically oriented epistemology is whether all necessary truth is "verbal." Even within the—fortunately ever increasing—camp of the analytic philosophers there is lively disagreement about the meaning of necessary truth, the connection between logical necessity and linguistic conventions, the precise meaning of analytic truth, the existence or even possibility of synthetic a priori propositions, etc. It is not even clear whether what is called "analytic philosophy" is an attempt to acquire a special kind of empirical knowledge, viz. knowledge about linguistic usage, or whether analytic philosophers are hunting after necessary truths—though by a method distinctly different from the methods of traditional metaphysicians—or, indeed, whether Schlick was right in saying that analytic philosophy is an activity of clarification of meanings which does not terminate in the assertion of distinctively "philosophical" propositions. However, it would be unwise to belittle an activity because those who are engaged in it are not clear about what they are doing. Great mathematicians may have nothing illuminating to say about the nature of mathematical proof, great physicists may be poor methodologists of physics; similarly it must be admitted that astounding progress has been made in analytic philosophy since the beginning of this century (the very best papers in this field written some fifty years ago may fail to meet the standards of acceptability imposed by leading journals of analytic philosophy today), although there is still great unclarity as to what philosophical analysis is. And there is nothing paradoxical about the fact that a great deal of clarification of concepts has been achieved with the help of the distinction between necessary and contingent propositions even though that distinction itself still needs clarification very badly.

The purpose of this book can be stated without ceremony, quite

will feel a strong urge to master it. Nevertheless, the improbable happens sometimes, and it is in order to help such brave uninitiated readers that I have appended a glossary of technical terms of logic and semantics that are used in the book. Had I defined all technical terms whenever they were first used so as to make them intelligible to laymen, it would be tedious reading indeed for professionals. The glossary, therefore, is a sort of compromise measure.

The bulk of this book was written, at intervals, between 1950 and 1953 at the University of Oregon, though I continued rewriting, as well as adding and deleting material, off and on until the fall of 1956. (Circumstances beyond my control prevented me from taking acount of more recent publications dealing with the problems of this book.) Since I spent the years at the University of Oregon in comparative isolation from analytic philosophers interested in the same problems, I must accept sole responsibility for either the virtues or the shortcomings of the book. Obviously I have been greatly influenced by several outstanding contemporary analytic philosophers in England and in America, but I find it impossible to say who influenced me most (though I would guess that Russell and Carnap have taught me more than all the rest). Above all, I owe a great debt to various brilliant students of mine whose questions and criticisms turned my teaching of analytic philosophy into a creative enterprise.

A few chapters were read and helpfully commented on by Professors A. F. Smullyan, University of Washington, and A. Kaplan, University of California at Los Angeles; I herewith extend my thanks to them. To Mr. David H. Horne of the Yale University Press I wish to express my appreciation for a careful and judicious job of editing the typescript.

Grateful acknowledgement is made to the following for permission to quote: the University of Chicago Press, for R. Carnap, "Logical Foundations of the Unity of Science," in *International Encyclopedia of Unified Science, I*, No. 1, 1938; the Columbia University Press, for Y. H. Krikorian, ed., *Naturalism and the Human Spirit*, New York, 1944; the Open Court Publishing Company, La Salle, Ill., for C. I. Lewis, *An Analysis of Knowledge and Valuation*, 1946; and the editor of *Philosophy of Science*, Vols. *3*, *18*.

ARTHUR PAP

New Haven, Connecticut
May 1957

CONTENTS

PART ONE

The Concept of Necessary Truth in Traditional Epistemology

SUMMARY OF PART ONE

IN Leibniz' philosophy we find a central preoccupation with the distinction between "truths of reason" and "truths of fact." This distinction does not, however, coincide with Kant's distinction between analytic and synthetic propositions, since "truths of fact," or even the broader class of empirical propositions, constitute according to Kant only a proper subclass of synthetic propositions. Although Leibniz, like Hume, did not explicitly distinguish the pairs a priori—empirical and analytic—synthetic, one may impute to him the theory that all a priori truth is analytic. In this respect he is, in spite of his usual classification as a "rationalist" (a word of most uncertain significance), an ancestor of contemporary logical empiricism. The disinclination to admit "intuition" as a source of a priori knowledge is, indeed, something which he has in common with logical empiricists of the present, though the latter are of course diametrically opposite to him in other respects. It seems, therefore, highly relevant to re-examine Leibniz' theory of a priori truth in a book which is centrally concerned with the epistemology of logical empiricism. It turns out that some of the difficulties in Leibniz' theory which Russell already brought to daylight in his early book on Leibniz (connected with the notion of "simple" concepts) reappear in Carnap's recent work in pure semantics and inductive logic.

Kant's definition of the analytic—synthetic distinction, and his doctrine of the synthetic a priori character of geometry and arithmetic, have been much criticized, and much of the criticism has been just. However, as against the tendency on the part of many of the logical empiricists who fought the battle against Kant (for which, undoubtedly, they are to be thanked) to relapse into a new kind of "dogmatic slumber," it is important to become aware of difficulties which modern analytic philosophy still has in common with Kant. They all turn, in my opinion, around the notion of analysis. Kant's definition of "analytic" is justly blamed for obscurity, since it speaks of "the" definition of a concept. But insofar as the analytic-synthetic distinction is of epistemological interest, a distinction must somehow be made between an arbitrary definition and a correct

analysis of a meaning antecedently entertained. And if we permit ourselves to use the notion of "correct analysis" without being able to analyze it, we should not be too contemptuous of Kant for having suffered from the same weakness. Again, Kant is accused of "psychologism," on the ground that the definition of "necessity" in terms of "what cannot be conceived, or imagined, to be otherwise" is couched in psychological terms whereas what is to be defined is a logical concept. But (as is shown particularly in Chapter 13 of this book) modern analytical philosophers are then equally "psychologistic" when they ask logically, or at any rate formally, undecidable questions of the form "Is p self-contradictory? Is q entailed by r?" in the attempt to judge proposed analyses of a concept. If we condemn such questions as meaningless, as some "systematic" analysts are inclined to do (see chapter 14, below), we even deny ourselves the right to affirm the logicist thesis of the reducibility of arithmetic to logic and hence the right to reject Kant's theory of arithmetic as demonstrably false.

Just as Leibniz is closer to logical empiricism than his usual classification as a "rationalist" would lead one to believe, so Locke is further from logical empiricism than a "British empiricist" would commonly be expected to be. In the first place, he explicitly distinguished within the genus of certain universal propositions, the instructive from the trifling ones, a distinction which corresponds almost exactly to Kant's distinction between those a priori propositions whose predicate is not contained in the subject and those whose predicate is so contained. And had he used the Kantian terminology he would have said not only that the axioms and theorems of Euclidean geometry are synthetic a priori propositions, but even that the true laws of nature, if only we had better evidence than the inductive kind, would be seen to be synthetic propositions about necessary connections (this interpretation will be textually substantiated in Chap. 3). Underlying this view is a conception of necessary truth which makes sense only if it makes sense to speak of apprehension of universals and of immutable relations between them—which should be enough to make a modern "empiricist" reluctant to look up to Locke as a congenial forefather. Let us not repeat forever the usual vague statement that, after all, Locke believed, unlike the rationalists, that "all knowledge is derived from experience"; for unlike the modern empiricists he did not

consider it a logical impossibility that a priori knowledge of synthetic propositions "about reality" be obtained. He held, indeed, that all ideas (to be distinguished from propositions) "come into the mind through the channels of sensation and reflection," but once this metaphorical statement is clarified by translation into the formal mode of speech, it turns out to be either false or tautologous: Locke did not want to maintain (no more than did Hume) that all descriptive predicates have meanings that can be grasped only by experience of instances to which they apply, but he wanted to maintain this with regard to simple descriptive predicates. Yet, he had no useful criterion of such simplicity except the causal/psychological one which makes his thesis analytic. This concept of simplicity is, however, of great importance for the theory of necessary truth, since one may be inclined to characterize as synthetic, necessary statements whose descriptive terms occur essentially yet cannot be eliminated through analysis.

In Hume we find the "psychologism" which is repudiated by the logical empiricists of the present who are in other respects more indebted to Hume than to any other big name in the history of philosophy: for his criterion of distinction between propositions "about matters of fact" and a priori propositions is in terms of what can or cannot be conceived to be otherwise. To be sure, he frequently speaks of "contradiction" when he raises the question (that powerful dialectic weapon against rationalistic pretenses of a priori demonstration) "Can p be supposed false without contradiction?" But since such questions can be decided only by "thought experiment," not by formal deduction, it is not clear that this formulation, preferred as it is by the modern analytic philosophers, is any less "psychologistic." Hume thus raised, by his own dialectic practice, a problem which verges into the problem of whether conceptual analysis can be so formalized as to dispense with the much decried appeal to the intuitive evidence of necessary connections—a question we examine and answer negatively at the end of this book.

CHAPTER 2. *Kant*

A. A Priori Knowledge and Necessity

KANT's explicit definition of a priori knowledge is a negative one: "knowledge that is independent of experience and even of all sense impressions" (*Critique of Pure Reason*, 2d ed., intro., I).[1] The kind of independence in question is not, of course, genetic, for Kant explicitly says that "undoubtedly all our knowledge begins with experience." What he had in mind is that a judgment is a priori if the *evidence* on which it is accepted is not empirical. This leaves us, of course, with "empirical" as an undefined term, but we must not deny to an epistemologist the privilege of taking some terms as undefined in order to be able to define others (the meaning of "empirical evidence" is, indeed, easily *illustrated*, by explaining, for example, that if the judgment "the straight line is the shortest distance between two points" were empirical, then it would be accepted on the evidence of repeated physical measurements of length). Kant proceeds to formulate a *criterion* in terms of which we can "infallibly distinguish pure (= a priori) knowledge from empirical knowledge":

> If we find, in the first place, a proposition which is conceived as *necessary*, then it is a judgment a priori; if, furthermore, it is not derived from any other proposition which is itself necessarily valid, then it is absolutely a priori. Secondly, experience never bestows on its judgments true or strict, but only supposed or comparative *universality* (through induction), so that we should properly say: as far as our observations go, there are no exceptions to this or that rule. If a judgment, then, is thought as strictly universal, i. e. in such a way that no exception at all is admitted even as a possibility, then it is not derived from experience, but is absolutely a priori [intro., II].

[1] All translations from the German in this chapter are mine.

Little analysis is needed to see that Kant's two criteria really coalesce into one, the criterion of necessity. For what does the contrast between "strict" and "only supposed" universality amount to? Kant surely does not mean that there are no universal empirical propositions that are true, i. e. that have no exceptions. All he means is that we never know with certainty that such a proposition is true, that there always remains the possibility of its being false. But then a "strictly" universal proposition is one which has no *conceivable* exceptions, which is another way of saying that it is necessary. We may confine our attention, therefore, to necessity as the touchstone of a priori knowledge.

If we call "subjective" a property of a proposition p which is such that to ascribe it to p is to say something about cognitive attitudes toward p, then there can be no doubt that necessity, and therewith a priori truth, in Kant's sense is a subjective property of propositions. To say that p is necessary is to say that p cannot be *conceived* to be false or is deducible from propositions that cannot be conceived to be false. Indeed, Kant explicitly says that in predicating a modality (such as necessity) of a judgment, one does not add anything to the "content" of the judgment but specifies the way in which the relation of the components of the judgment (subject and predicate, or antecedent and consequent) is conceived.[2] As we shall see, the attempts at explication of the concept of necessary truth which followed the Kantian era are characterized precisely by the ambition to de-psychologize, if I may coin a word, this concept, and one might say that to this extent their *Leitmotif* was "back to Leibniz!" It must be said, however, in all deference to the genius of Kant, that while "p cannot be conceived to be false, or is deducible from propositions which cannot be conceived to be false" seems to be the primary meaning he

[2] Kant's own formulation is somewhat obscure: "The modality of judgments is a very special function of theirs, whose distinguishing characteristic is that it adds nothing to the content of the judgment (for besides quantity, quality, and relation nothing is left as part of the content of the judgment), but concerns only the value of the *copula* in relation to thought" (*Critique of Pure Reason*, Transcendental Doctrine of Elements, Analytic of Conceptions, chap. 1, sec. 2). Especially is it not clear what he means by "content": "p is possible" and "p is necessary," which are forms of positive modal judgment, surely are not equivalent, and in that sense do not have the same content. I think, however, his intention was to say that a modal judgment is about cognitive attitudes.

attached to "p is necessary," it is impossible to make Kant out as *consistent* in his usage of this central term. When he says that it is a necessary (or a priori) truth that two straight lines cannot enclose a space, or that the straight line is the shortest distance between two points, he is clearly referring to the impossibility of imagining an exception. However, a serious deviation from the specified meaning can be spotted in Kant's discussion of causality. In the very introduction to the *Critique of Pure Reason* where the difference between empirical and a priori knowledge is explained, Kant cites the principle of causality, that every change has a cause, as an example of a necessary proposition. In support of his claim, made in criticism of Hume, that this is a *necessary* proposition he says: "indeed, in the latter (the proposition "every change has a cause") the very concept of cause contains so evidently the concept of a necessary connection with an effect and of strict universality of the rule, that it would become entirely unrecognizable if one wanted, following Hume, to derive it from a frequent conjunction of an event with a preceding event and the resulting habit (and thus merely subjective necessity) of association of ideas." What Kant maintains here is that the concept of necessary connection is indispensable for an adequate formulation of the principle of causality, thus: "for every change there is an antecedent event which is necessarily connected with it." (The serious ambiguity, insufficiently attended to by both Kant and Hume, that this might mean "for every event there is an antecedent which is necessarily followed by the event" or "for every event there is an antecedent which necessarily precedes the event" need not detain us in this context.) But the principle, thus formulated, does not entail that *it is necessary* that for every change there is an antecedent event which is necessarily connected with it; in other words, it is perfectly compatible with the proposition, maintained by Hume, that it is conceivable that there should occur a change which is uncaused in the sense that there is no antecedent necessarily connected with it. It is, indeed, unlikely that Kant intended to maintain the intuitive inconceivability of an uncaused change in the sense in which he maintained the intuitive inconceivability of a space that did not conform to the propositions of Euclidean geometry. As a matter of fact, the neo-Kantian interpreters of Kant have emphasized that Kant held the "principles of experience," of which the principle of causality

is one, to be necessary in the sense of being *necessary presuppositions* of empirical science. This, however, is a complete shift of meaning of the term "necessary": from the fact that acceptance of proposition p is a *sine qua non* for the pursuit of inductive science, in the sense that from his very use of scientific method one can infer that the scientist believes p, it does not follow that p is a necessary proposition in the sense that it cannot be conceived to be false.

The poverty of the cited argument for the necessity of the principle of causality is clearly revealed if we consider that the same argument, if consistently employed, would have forced Kant into contradiction with his explicit admission that *specific causal laws*, unlike the principle of causality, are contingent propositions. For in line with his rejection of Hume's contention that the alleged necessary connection between cause and effect consists only in a "subjective necessity" (the pressure of the "gentle forces of association") he held that the concept of "objective necessity" is involved in a specific causal law like "the heat of solar radiation causes a block of ice to melt" just as it is involved in the general principle of causality. But then this specific causal law would have to be held to be a necessary proposition if the fact that the concept of necessary connection is a constituent of proposition p were a sufficient reason for holding p to be necessary. But quite apart from this consideration, the mentioned shift of meaning of the word "necessary" can be clearly traced in Kant's famous "proof" of the principle of causality in the section entitled "second analogy (of experience)." Kant observes that without the concept of causal order it would be impossible to distinguish objective and subjective temporal order of events. Mere perception, he says, is unable to determine the *objective order* of successive phenomena. For example, I might at this moment hear a voice and the next moment see the person whose lingual movements caused the sound; the effect, the sound, is perceived first, and the cause, the lingual movements, second. If I understand Kant (which is not easy), he is saying that in judging the lingual movements as the objectively earlier event I implicitly make the causal judgment that the lingual movements are the cause and the sound the effect. At any rate, Kant does maintain that if there were no causal order among a series of events $e_1, e_2 \cdots e_n$, then their *real* temporal relations would be indeterminate, it would be arbitrary whether we say e_1 is earlier

than e_2 or e_2 earlier than e_1 or e_1 simultaneous with e_2. How he could maintain this in view of the (I should think) undeniable fact that we often agree that one event *really* preceded another event yet is causally unrelated to the latter, and that the proposition agreed upon is surely not self-contradictory (if it were, temporal sequence would be indistinguishable from causal sequence), I leave to the profounder Kant scholars to decide. For the sake of the argument, let us concede that the proposition that there is an objective temporal order of events entails the principle of causality. It is clear that this would amount only to a proof of the necessity of the proposition "if there is a real (objective) temporal order, then every event has a cause," but not to a proof of the necessity of the consequent of this conditional. In other words, what Kant would have proved at best is that the principle of causality is *necessarily presupposed* by the belief in an objective temporal order. He would not have established that a world devoid of objective temporal order—i. e. a world in which sometimes the impact of the stone is followed by the breaking of the window and sometimes follows the breaking of the window, in which a state of nonuniform density of a gas is sometimes followed by a state of uniform density and sometimes the reverse sequence occurs—is *intuitively inconceivable.* He is thus guilty of equivocation upon the term "necessity."

The central point to be kept in mind is that Kant's explicit distinction between the concepts of necessary and analytic truth made it impossible for him to adopt Leibniz' apparently nonpsychological criterion of necessary truth, the self-contradictoriness of the negation (in other words, the *logical* impossibility of exceptions). The alternative criterion which accommodates analytic truths as a subclass of necessary truths inevitably employs the wider concept of *inconceivability* (whether or not it be called "psychological"), for the word "possible" in Kant's statement "no exceptions (to an a priori truth) are admitted as possible" cannot mean what it meant for Leibniz: "consistent with the law of identity (or the law of contradiction)." However, if it were not for the tacit shift from "cannot be conceived to be false" to "necessarily presupposed by what is claimed as knowledge of objective reality," the extension of the term "necessary truth" would have been far smaller for Kant than he claimed it to be, and much of his verbal disagreement with Hume would never have arisen.

B. *The Definition of "Analytic"*

It has often been pointed out that Kant's definition of an analytic judgment as a judgment whose predicate is (implicitly) contained in the concept of the subject (see *Critique of Pure Reason,* intro., IV) is unsatisfactory, not only because the literal meaning of the metaphor "contained" is not clear, but also because ever so many judgments (or propositions, as we say nowadays) do not have subject-predicate form and yet the analytic-synthetic division was intended by Kant as exhaustive with respect to the class of true propositions.[3] Let us first give our attention to the latter limitation. That Kant should not have been aware of it is particularly surprising since the table of the twelve different forms of judgments ("logical functions of the understanding") played such an important role in his doctrine of the categories. Take, for example, negative judgments, like "no triangle has four sides." Kant surely would have classified it as analytic, yet the predicate is so far from being contained in the subject that it contradicts the subject; the cited definition of "analytic," therefore, is restricted not only to judgments of subject-predicate form but even to affirmative judgments of that sort.[4] Again, existential judgments, like "there are cows," would no doubt have been classified as synthetic by Kant, yet the only way they could be construed as having subject-predicate form would be by treating existence as a predicate, which would be contrary to Kant's own famous thesis that existence is not a predicate. Again, consider hypothetical judgments, like "if somebody is somebody's teacher, then somebody is somebody's pupil" (which is reducible to a substitution-instance of a theorem of the logic of relations): Kant himself, in discussing the table of logical forms of judgments, mentions the relation of antecedent to consequent as distinct from the relation of subject to predicate; indeed, if the antecedent were construed as the "subject," then the expresssion "concept of the subject" would become unintelligible.

[3] *Self-contradictory* propositions, of course, are neither analytic nor synthetic in the Kantian senses of these terms; for this reason I characterize the divided class as the class of *true* propositions.

[4] Cf. on this point, Marc Wogau's illuminating study "Kants Lehre vom Analytischen Urteil," *Theoria* 1951, Pts. I-III. Kant lifted, though, this particular restriction in the *Prolegomena,* § 2: "the predicate of an affirmative analytical judgment is already contained in the concept of the subject, . . . In the same way its opposite is necessarily denied of the subject in an analytical, but negative, judgment."

of analytic truth a criterion which at least is not obscurer than the concept of logical truth itself.

Let us now turn our attention to the question of what exactly is the relation between subject and predicate in an analytic judgment according to Kant. In the *Prolegomena to Any Future Metaphysic* Kant improved on the cited definition from the *Critique* insofar as he dropped the metaphor " contained ": "Analytical judgments express nothing in the predicate but what has been already actually thought in the concept of the subject, though not so distinctly or with the same (full) consciousness." The psychological language here used has been the focus of much criticism. For example, it has been said that the connotations of a term vary from individual to individual, and that therefore Kant's concept of analyticity is psychological. (It makes sense to say that " *S* is *P* " is analytic for so and so, but not to say simply " ' *S* is *P* ' is analytic.") Consider, for example, Kant's claim that " all bodies are extended " is analytic while " all bodies have weight " is synthetic; just how would Kant prove, so the objection runs, that people are not thinking of weight when they think of a body? Kant would, of course, admit that, by virtue of what Hume called " habits of association," the thought of a body is *accompanied by* the thought of weight, just as the thought of a dog might be accompanied by the thought of barking. But he would deny that having weight is part of the *meaning* of " body." And he would, had he been reared in the language of contemporary analytic philosophers, support this claim by appeal to the fact that the concept of a weightless body (unlike that of a body devoid of inertia) is *not self-contradictory*; or that " *x* is a body " does not analytically *entail* (though it may factually imply, i. e. imply by virtue of an empirical law) " *x* has weight." If Kant's conception of analyticity, then, is to be condemned as " psychologistic," at least he will enjoy the company of many subtle contemporary analysts of reputation. When we argue nowadays " it does not seem self-contradictory to suppose that a body existed all alone in space, and since weight is a relation of a body to other bodies (consisting in its being attracted to other bodies), it follows that weight is not part of what is *meant* by ' body,' " we equally rely on " thought experiments."

The problem of philosophical semantics which is implicit in Kant's statement about the relation of subject and predicate in

analytic judgments is simply the problem of what a suitable criterion of *identity* (total or partial) *of concepts* might be.[7] That Kant failed to solve this problem is surely a forgivable sin if one considers that the entire problem of the identity conditions of intensions (when are two properties identical, when are two propositions identical?) is still highly controversial nowadays in spite of the professed rejection of " psychologism " in philosophical semantics. In his study referred to above, Marc-Wogau concentrates on just this difficulty with Kant's doctrine of the analytic judgment. Consider the judgment " every triangle has three angles." It would not help to say that " having three angles " is a constituent of the concept " triangle " if and only if it occurs in the *definition* of " triangle," for thus the burden would merely be placed on the question " what is meant by *the* definition of a concept? " We might define " triangle " to mean " plane figure bounded by three straight lines and having three angles "; in that case the judgment would be analytic by the above criterion of partial identity of subject and predicate. But it would commonly be said that such a definition involves a redundancy: " plane figure bounded by three straight lines " is sufficient, one would say, since the other property is *deducible* from this definition. But in what sense is it " deducible? " The situation is not quite analogous to the redundancy in the definiens for " square ": equilateral, four-sided rectangle; for here the redundant predicate " four-sided " can be extracted simply by defining " rectangle " (" four-sided rectilinear figure all of whose angles are right angles "), while " triangular " is not in the same straightforward sense contained in the definiens of " trilateral." Perhaps a formal deduction of " *x* is triangular " from " *x* is trilateral " requires the theorem " if a closed figure has *n* sides, then it has *n* angles," but if " *P* is contained in *S* " is used in the sense of " *P* is deducible from the definition of *S* together with axioms or theorems of the system in which *S* is defined," then we obtain of course a concept of analyticity which is far wider than the one Kant had in mind (" all triangles have as the sum of their angles 180 degrees " could be analytic in that sense!) and which, moreover, turns " analytic " into a term relative to a deductive system, which was definitely not Kant's intention. Kant, then, would be hard pressed were he asked whether the concept " triangle " is adequately

[7] See below, Chap. 10.

defined as "rectilinear figure with three sides and three angles" or as "rectilinear figure with three sides" or as "rectilinear figure with three angles."[8]

Since the interpretation of "*P* is contained in *S*" as meaning "*P* occurs either directly or indirectly—through expansion of definienda into definientia—in the definiens of *the definition* of *S*" comes nearest to giving an objective meaning to Kant's term "analytic," it is with considerable curiosity that we look to Kant's own statements about the nature of definitions.

In the *Critique* we read, in the section entitled "The discipline of Pure Reason in its Dogmatic Use" (Transcendental Doctrine of Method, chap. I, sect. I): "As the very term 'to define' indicates, to define means nothing more than to express the complete concept of an object within its limits and in underived manner." The explanations of the terms "complete" and "within limits," given in a footnote, make it clear that Kant meant to say that the definiens must express, in clear language, a necessary and sufficient condition for applicability of the concept, and must contain no redundancy; and by the requirement of "Ursprünglichkeit" is meant, as he explains in the same footnote, that the defining property should not stand in need of demonstration. It is obvious that Kant had the Aristotelian distinction between essence and property in mind, according to which "a plane figure with the sum of its angles equal to 180 degrees" could not serve as definiens for "triangle" because, even though it is convertibly predicable of triangles, it does not express the *essence* of triangularity but rather "flows" from the latter. And since to clarify Kant's concept of analyticity is the same as to clarify the expression "essence of a concept," this explanation gets us nowhere; in fact, the notion of essence is made no clearer by Kant than it was by Aristotle himself. In his *Logic* (*Vorlesungen zur Logik*) he makes the traditional distinction between *nominal* and *real* definitions (§ 106). But it is simply impossible to make any consistent sense of his disconnected remarks on the distinction.[9] A nominal definition, we are told, is an arbitrary stipulation of a meaning for a given name, while a real definition demonstrates

[8] Amusingly, as Marc-Wogau brings out, Kant was inconsistent in his claims as to what is to be regarded as the definition of "triangle." See *loc. cit.*, p. 151.

[9] And since Kant authorized the publication of these lecture notes, we cannot hold his students responsible for this blemish.

the possibility of the defined object from its "inner" marks (I have nowhere found as much as a hint of the meaning of "inner"). Real definitions, we are told, are to be found in mathematics, "for the definition of an arbitrary concept is always real." What Kant elsewhere says about mathematical concepts leaves no doubt that "arbitrary concept" here means "constructed concept." The mathematician, Kant held (following Locke, whose distinction between ideas of substances and ideas of modes must have influenced him more than he liked to admit),[10] does not abstract his concepts from empirical objects but *constructs* them prior to experience of instances. But consider, then, the definition of "ellipsoid" as meaning "solid generated by rotating an ellipse around either one of its axes," where "ellipse" is similarly given a genetic definition, viz. "closed curve all of whose points are such that the sum of their distances from two fixed points is constant" (this definition is usually called analytic but Kant would have called it "genetic" because we can derive from it a recipe as to how an ellipse might be constructed). This is undoubtedly the sort of thing Kant had in mind when speaking of the real definitions of constructed mathematical concepts which guarantee the possibility of the defined object since we can, following the recipe, construct on the blackboard or on white paper objects satisfying them. Yet if the definiendum "ellipsoid" has no antecedent usage and has just been invented as an abbreviation for the complex definiens, then the same definition is nominal according to the explanation given! The carelessness of Kant's thinking (or at least lecturing) on the nature of definitions is sufficiently illustrated by this point, and we must therefore conclude that to the extent to which the meaning of Kant's "analytic" depends on the meaning of "*the* definition of the subject-concept," it is obscure.

Before leaving the subject, I would like to call attention to a most interesting observation made by Marc-Wogau, connected with Kant's statement that, strictly speaking, concepts of natural kinds, like gold, cannot be defined at all:

> For, as an empirical concept consists only of a few marks of a certain kind of object of the senses, it is never certain whether one might not mean by a word denoting an identical object at

[10] Cf. the self-conscious confession of indebtedness to Locke in the *Prolegomena*, § 3.

> one time more, at another time fewer marks of the object. Thus one person's concept of gold might contain, besides the weight, the color and the solidity, the property of being uncapable of rusting, while another person may not know this property. A fixed set of marks is used only as long as they suffice for the purpose of distinguishing the kind from others; through new discoveries, however, some marks get removed and some get added, and consequently the concept is never perfectly fixed [*Critique of Pure Reason, loc. cit.*].

Marc-Wogau comments that thus the sentence "gold does not get rusty in water" is analytic for the first person, synthetic for the second. Similarly, an identity sentence of the form "a = the x with property P" (where "a" is a proper name) might be said to be analytic for a person in whose usage "a" is precisely an abbreviation for that description, and synthetic for a person using the same proper name as abbreviation for another description which, though denoting the same object as the first, is based on a predicate Q which is not synonymous with P. However, is all that Kant's apperçu amounts to that the same class term may be given different definitions by different people, such that instead of saying "S is analytic" we ought to say "S is analytic *as used* by X"? It rather seems that Kant saw, though none too clearly, that statements about natural kinds, like "gold is yellow," cannot be classified as analytic or synthetic in the sense in which statements about mathematical objects are so classifiable, for the reason that "analytic" was defined in terms of "definition," and there can be no "definitions" of natural kinds in the same sense of the word as there can be definitions of mathematical concepts. To be sure, Kant did not clearly explain *why* concepts of natural kinds should not be "strictly definable." But perhaps he could have made his point as follows. Suppose that "gold" were defined as a yellow metal with a definite atomic weight and a definite melting point. If this were a "definition" in the same sense in which "'square' means 'equilateral rectangle'" is a definition, then it would be self-contradictory to classify an object which had all the defining characteristics except the color as gold, just as it would be self-contradictory to classify, say, a rhombus as a square. If a scientist observing such an anomalous specimen insisted, "Still, this *is* gold, so we must recognize that not

all gold is yellow," he would have to be interpreted as recommending a redefinition of "gold," not as pronouncing an empirical generalization refuted. But if he said instead, "This is not a species of gold, it's a different species though closely similar to gold," this would likewise be an acceptable comment. Just which way the discovery of the goldlike, hitherto unknown, specimen will affect his classification of natural kinds will depend on pragmatic considerations. Now, Kant might hold that no analogous situation could occur in connection with geometrical and arithmetical concepts, that there could be no occasion for "redefining" such concepts in the light of new discoveries about their instances; and that this suggests that "definition" does not have the same meaning in "definition of 'gold'" as in "definition of 'square.'"[11]

C. *Synthetic A Priori Truth in Geometry*

Of all the synthetic a priori propositions alleged as such by Kant, those that illustrate his conception of an a priori truth as a universal proposition exceptions to which are *inconceivable* most clearly are geometrical axioms, like the famous axiom "two straight lines cannot enclose a space." In fact, if one were pressed to explain the relevant meaning of the word "inconceivable," one might well do it denotatively by giving such examples as the inconceivability of a space enclosed by two straight lines. As to the *intuitive* nature of geometrical knowledge Kant made two claims: first, that our knowledge of the axioms is intuitive ("intuitive" being contrasted with both "empirical" and "analytic"), and secondly, that even the deduction of theorems from the axioms requires spatial intuition. The latter claim was not any more far-fetched than the former, considering the role played by *constructions* in the proofs of Euclidean geometry such as the proof of the theorem about the sum of the angles of a triangle (see particularly the *Critique*, "Transcendental Doctrine of Method," chap. 1, sec. 1). However, Kant here failed to make the distinction, often urged nowadays, between the context of *discovery* and the context of *justification*. If it were granted that constructions are indispensable for the discovery of proofs, it still would not follow that recourse to constructions is required for validating a proof once discovered. Hilbert has shown that if only

[11] We shall return to this problem later. See below, pp. 112-16.

all the axioms tacitly assumed by Euclid in his proofs are made fully explicit, then purely *formal* proofs of the theorems can be given. We shall concentrate, accordingly, on the former claim, of the inconceivability of the falsehood of the Euclidean axioms. It has been exposed to heavy fire since the publication of non-Euclidean systems of geometry discredited Kant's doctrine of the finality of Euclidean geometry. We shall begin with the question of whether the axioms are a priori, and then turn to the question whether they are synthetic.

A rather naive argument against the claim of self-evidence for an axiom like "two straight lines cannot enclose a space," which is nevertheless often advanced, is that this proposition has even been shown to he false, since it does not hold in Riemannian geometry; which geometry is actually suited for the description of physical space if only we consider sufficiently large areas. Now, it is clear that if a system of pure geometry contains the contradictory of the sentence "a straight line is uniquely determined by two points," the term "straight line" does not in that system mean what it means in the system containing the contradicted sentence. For to the extent that the primitives "point," "straight line," "plane" have any meanings at all as part of such an uninterpreted system of geometry, they mean whatever entities satisfy the axioms, and the same entities cannot satisfy mutually contradictory axiom sets. For example, Riemannian "straight lines" can be interpreted as great arcs on a spherical surface, and with this interpretation it becomes true to say that two straight lines may enclose a space. But it is not then the *proposition* expressed by the sentence "two straight lines cannot enclose a space" in the Euclidean system which has been contradicted. Those who are anxious to defend the thesis that all a priori truth is analytic may seize the opportunity to point out that what on this analysis is an a priori truth is the proposition that two *Euclidean* straight lines cannot enclose a space, which is analytic because "Euclidean" can be defined only in terms of satisfaction of just such axioms as the one in question. But discussion of the tricky question whether the axiom is analytic or synthetic had better be postponed until careful attention has been given to the question of whether any of the arguments against the *necessity* of the axiom are sound.

A well known objection—one, indeed, which people are apt to

echo in rather thoughtless manner just because it is "classic"—is that inconceivability is altogether relative to experience and accepted scientific theory. It is said again and again that the history of science amply proves that what is self-evident to one generation is rejected as false by the next generation on the authority of well-confirmed scientific theory. And the classical example of this is the question of the conceivability of antipodes. Thus John Stuart Mill, in the context of arguing for the empirical character of geometrical truths, writes:

> There are remarkable instances of this in the history of science; instances in which the most instructed men rejected as impossible, because inconceivable, things which their posterity, by earlier practice and longer perseverance in the attempt, found it quite easy to conceive, and which everybody now knows to be true. There was a time when men of the most cultivated intellects, and the most emancipated from the dominion of early prejudice, could not credit the existence of antipodes; were unable to conceive, in opposition to old association, the force of gravity acting upward instead of downward [*System of Logic* (1887), Bk. II, chap. 5, § 6].

But this argument, plausible as it sounds, is spoiled by an ambiguity of the word "conceivable." That people on diametrically opposite points of a sphere should both remain attached to the sphere without any tendency to "fall off" is inconceivable in the same sense in which it is inconceivable that a person could walk on the surface of a lake, or that a person could jump off a cliff and find himself rising instead of falling. Here "inconceivable" means "unbelievable" but not "unimaginable." Experience has developed in us an irresistible tendency to expect A to be followed by B, but if we did not admit that an exception is in some sense imaginable, it would be difficult to explain what we mean by saying that "A is always followed by B" is a *contingent* (or *empirical*) truth. That people ever found the existence of antipodes unimaginable in the sense in which a space enclosed by two Euclidean straight lines, or a space of four dimensions (in the ordinary, not the generalized, sense of "space"), is unimaginable, is a wildly unplausible assumption; at any rate, the historical evidence that they declared this sort of thing "impossible" or "inconceivable" is not relevant, since

we declare many things impossible or inconceivable which we find it impossible to *believe* but which we can *imagine*, or *think of*, without excessive difficulty. Who could believe that there is a man who will never die? But who has serious difficulty in forming the concept of an immortal man, or even of a man whose appearance remains unaltered during a thousand years?[12]

Curiously, Mill admits that once we have acquired through experience with straight lines the notion of straightness, mere reflection upon this notion suffices for revealing the truth of the axiom. He approvingly quotes Bain's statement (as going "to the very root of the controversy"): "We cannot have the full meaning of Straightness, without going through a comparison of straight objects among themselves, and with their opposites, bent or crooked objects. The result of this comparison is, *inter alia*, that straightness in two lines is seen to be incompatible with inclosing a space; the inclosure of space involves crookedness in at least one of the lines" (*System of Logic*, § 5). But if the fact that reflection on the meaning of a sentence *S* is sufficient to produce assent to *S* does not prove that *S* expresses an a priori truth, what could "a priori truth" mean? Mill may have been right in holding, following Locke and Hume, that such geometrical concepts as straightness are derived from sense impressions of straightness (and in that sense are empirical concepts), but it surely is possible for a necessary proposition to contain empirical concepts. It appears, then, that it is rather obscure what Mill was denying when he denied that the axiom in question is a *necessary* truth. And similarly we must challenge anyone who, while admitting that enclosure of a space by two straight lines cannot be imagined in the sense in which dogs that can fly and speak French can be imagined, denies that the axiom is a necessary truth, to explain what he means by a "necessary truth."

There are those who hold that if the negation of a proposition is inconceivable in an absolute sense, i. e. not just in the sense

[12] Unfortunately Whewell, whose doctrine of the necessity of geometrical axioms Mill attacked, was himself guilty of this equivocation on the term "inconceivable" (as meaning sometimes "unimaginable" and sometimes "unbelievable"), and for this reason Mill's criticism is much more persuasive than it would otherwise be. Thus Mill cites with considerable glee Whewell himself as reporting that such revolutionary theories as the Copernican, and Galileo's theory of uniform motion as not requiring to be sustained by force, were at the time rejected as inconceivable.

that habits of association produced by experience make it *psychologically* impossible to believe it, this can only be because the proposition is definitionally true, "analytic." Since "*p* is definitionally true" means presumably that *p* is deducible from logical principles (such as the principle of identity) with the help of adequate definitions, it would indeed be difficult to explain the absolute inconceivability of the negation of a logical principle in the same way.[13] But be this as it may, let us see whether Kant can be successfully refuted by proving that such a geometrical axiom is analytic. We may formulate the axiom as an axiom of plane geometry as follows: two straight lines have either no point in common or else exactly one. When Kant pronounced it as "synthetic" he could hardly have had in mind his definition of a synthetic judgment as one whose predicate is not contained in the subject, for if there are any propositions which do not have subject-predicate form, this is one. Let us, then, take "analytic" in the sense Kant must have intended when he said that the principle of contradiction is the sufficient ground of analytic judgments: the judgment cannot be denied without self-contradiction. The difficulty faced by the claim that our axiom is analytic in this sense is that the only explicit definition of "straight line" which would enable a proof of analyticity is one which Kant would have regarded as itself a synthetic axiom: a straight line is a line which is uniquely determined by two points (i. e. given two points, there is one and only one straight line containing both).[14] Given this definition, the axiom could indeed be formally demonstrated: if two straight lines did have two points in common, then there would be two distinct straight lines containing two given points, which contradicts the definition of "straight line." The fact that a definition which seems perfectly adequate could be rejected by a Kantian as question begging[15]—since on the same general grounds on which he regards the axiom to be demonstrated as synthetic he would hold the definition to express a synthetic judgment—is indeed a forceful

[13] On the question whether logical principles can significantly be said to be "analytic" in the above sense, see below, Chap. 5.

[14] Kant mentions somewhere that the proposition "three points uniquely determine a plane" is synthetic, hence one would expect him to hold the same view with respect to the analogous proposition about the straight line.

[15] The problem of definitions begging the question of analyticity, here touched upon, will be discussed in some detail below, Chap. 8.

reminder that, unless one can clarify the relevant sense of "adequate definition," the concept of analyticity is hopelessly obscure. But let us see whether there is some way in which a Kantian could be forced to surrender. One might argue that since "straight line" has different meanings in different systems of geometry, we have to fix the relevant concept by specifying a definite kind of plane, thus: "straight line in a Euclidean plane," "straight line in a Riemannian plane," etc. We would then be led to the question of what is meant by "Euclidean plane." Now, a type of plane, so we might argue, can be defined in no other way than as the type of plane for which the axioms of a given plane geometry hold. More precisely, the idea is this: we formulate the axioms "two straight lines in *P* have either no point or exactly one point in common," "given a point in *P* and a straight line in *P* not containing that point, there is exactly one straight line in *P* which contains that point and has no point in common with the given straight line," and others. So far the axioms are propositional functions, neither true nor false, since they contain the variable "*P*"; hence it would not make sense to ask whether they are analytic or synthetic. But the only way "*P*" could be given a meaning that guarantees the truth of the axioms would be by the definition "*P* is whatever satisfies these axioms." Now the axiom will read "two straight lines in a plane in which two straight lines have either no point in common or exactly one point in common have either no point in common or exactly one point in common," which is surely analytic. But the trouble with this trick is that it leads us into a circle. For a type of plane has been defined in terms of "straight line," but our starting consideration was that "straight line" is ambiguous until a definite type of plane is specified.

It is difficulties of this sort that presumably led such mathematicians as Hilbert to abandon the quest for *explicit* definitions of the terms "straight line," "point," "plane," and instead to define them implicitly as whatever entities satisfy the formal axioms formulated by means of them. The Kant critique since Hilbert accordingly took the form that the axioms of *pure* geometry are neither analytic nor synthetic, for they are propositional functions, not propositions. The question arises of whether this may be regarded as a refutation of Kant, and this is to ask whether Kant meant by *pure* geometry what is nowadays meant by it. The question answers itself: Kant, after all, lived *before* Hilbert. But is it possible to make the meaning of "pure geometry" intended by *Kant* intelligible?

It seems clear that the Kantian concept of "pure geometry" can be defined only in terms of the notion, going back to Plato, of *a priori concepts* ("innate ideas"). An a priori concept would have to be defined as a concept which, though it is not purely formal (like the logical constants, "not," "implication," etc.), is neither ostensively definable nor analytically definable in terms of ostensively definable concepts. That Kant thought of "straight line" as just such an a priori concept cannot be doubted by anyone who is aware of the impression which Plato's theory of innate ideas (whatever its cognitive content may be) must have made on Kant via Descartes. Kant would say that no instance of a perfect straight line could be given in sense perception ("empirical intuition"); hence ostensive definition is out of the question. But I think that he moreover held the concept to be unanalyzable, a *simple* concept. For his proof of the synthetic character of the axiom "the straight line is the shortest distance between two points" is that the subject-concept is purely qualitative while the predicate is quantitative. Yet any conceivable analysis of "straight line" would have to be in what Kant would call "quantitative" terms: "class of points which is uniquely determined by *two* points" is quantitative too! But by the same argument Kant should even have denied that the statement "the circle is a closed curve all of whose points are equidistant from a given point" is analytic: if we understand what Kant means by saying that straightness is a *quality*, we must surely confess that circularity is in the same sense a quality. Now, if Kant were asked why on earth he rejects such definitions as not really *analyzing* the subject-concept, he would, I think, make the reply that you could not teach someone the meaning of the terms "straight" and "circular" by means of such quantitative descriptions unless he had, stimulated by sense experience, already acquired these concepts. If so, then Kant implicitly used the expression "analysis of concept *C*" in such a way that a given definition could be said to express an analysis of *C* only if it is possible to make a person who had never experienced an instance of *C* think of *C* by producing that definition. Thus "color between yellow and blue" would not express an analysis of the concept *green* if it is impossible to make

a person who has not seen green before think of green by means of that description.[16]

Supposing that "straight line" and "point," as used in pure geometry in Kant's sense of "pure," express such simple a priori concepts (dispositions, perhaps, to remember intellectual visions enjoyed in Plato's heaven of Forms), what follows with regard to the question of whether the axioms of such a pure geometry are analytic or synthetic? If "S is analytic" means "S is deducible from logical truths *with the help of adequate analyses* of the terms of S," and the relevant terms of S are not analyzable at all, does it then follow that S is synthetic? It seems to me that the question does not admit of a nonarbitrary answer, because philosophers have not taken the trouble to specify whether the statement "S is *not* deducible from logical truths with the help of adequate analyses" is to be taken as true in case no adequate analyses of the relevant terms can be produced at all. It is somewhat like the question of wbether a man who never beat his wife has or has not stopped beating her. In fact, as we have seen, Kant himself raised the question of what sorts of concepts are definable at all—though he sadly neglected to clarify the relevant meaning of "definable"—and maintained that concepts of natural kinds, for example, are not definable. This did not prevent him from declaring elsewhere that "gold is yellow" is an analytic judgment. In the same way, his implicit view that "straight line" is not definable did not prevent him from declaring that "the straight line is the shortest distance between two points" is *not* analytic. What he did not consider was the possibility that the division analytic-synthetic is not significantly applicable to statements whose relevant terms are unanalyzable.[17]

[16] We shall return to this criterion of complex, or analyzable descriptive concepts in Chap. 9.

[17] Although a suitable restriction of the class of statements to which the analytic-synthetic dichotomy applies would, as we shall see again later, spare philosophers some embarrassment, no such restrictions are usually considered by philosophers operating with the dichotomy. In particular, the supposed unanalyzability of the relevant predicates in a given statement is usually regarded as a basis for classifying the statement as synthetic. Thus ethical intuitionists who hold ethical predicates like "right" and "good" to be unanalyzable pronounce statements connecting such predicates with naturalistic predicates for this very reason as synthetic.

D. Synthetic A Priori Truth in Arithmetic

If Kant's proof of the synthetic character of the axioms of pure geometry leaves much to be desired—mainly, as we have seen, because of the unclarity of his analytic-synthetic distinction—this holds to an even higher degree of his proof that judgments like "$7 + 5 = 12$" are synthetic. His statement in support of this conclusion, viz. that by merely thinking of the sum of 7 and 5 we cannot discover that it is equal to 12, can only mean that this truth is not *self-evident*. But had Kant given only a minimum of thought to the implied claim that all analytic truths are self-evident, he would have disavowed it. By a series of definitions of the form $n = m + 1$, any true equation of the form "$x + y = z$" (where the variables stand for integers) can be reduced to an identity; since the denial of such an equation would, therefore, be equivalent to a violation of the law of identity, Kant would surely have to admit that these equations are analytic by his own criterion of analytic truth; yet if all such equations were self-evident, there could be no need for *learning* addition.

However, when Kant stated that the principle of contradiction is the sufficient ground of all analytic truth, he overlooked like his predecessor Leibniz that without additional axioms which are not reducible to identical propositions formal demonstration could hardly take a step. Thus associative, distributive, and commutative laws of addition and multiplication are presupposed in the proofs of theorems of arithmetic and algebra; and in proofs by mathematical induction the principle of mathematical induction is presupposed. Now, a staunch and stubborn defender of Kant might argue that Kant was right after all, for this reason: if the axioms from which a theorem is formally deduced are synthetic, so is the theorem, and it would be a mistake to suppose that since the *implication* from axioms to theorem is analytic the theorem itself is analytic. And here he might quote Kant's own words:

> All mathematical judgments are synthetic. This proposition seems to have escaped the notice of the analysts of human reason up to date, indeed it seems to go against all of their opinions even though it is incontestably certain and of great consequence. For as they found that all the deductions made by mathematicians proceed in accordance with the principle of

contradiction (which is required by the nature of apodeictic certainty), they supposed that the axioms themselves are known on the basis of the principle of contradiction; which is an error: for it is indeed possible to prove a synthetic proposition by the principle of contradiction but only by presupposing another synthetic proposition from which the former is deducible, yet never in itself [*Critique*, intro., V].

This, however, is a poor defense of Kant. For as we have amply illustrated in the discussion of Leibniz, if the use of axioms other than the principle of identity in the demonstration of a proposition entails that the proposition is *not* analytic, then demonstration of an analytic proposition becomes almost a logical impossibility. Such a narrow definition of "analytic" would in fact entail that principles of formal deduction, like the principle of the hypothetical syllogism (if p implies q, and q implies r, then p implies r), are synthetic [18]; yet Kant characterized logic as a purely formal science in which pure intuition is not operative. Indeed, it is likely that Kant never reflected on the question of whether such an axiom of arithmetic as the associative law of addition is analytic or synthetic, and had he reflected on the question he might have become painfully aware of the inadequacy of the definitions of "analytic" which he offered.

When Frege, about a century after Kant, re-examined the question of the nature of arithmetical truth, he based his discussion on a far more careful and acceptable definition of analytic truth than Kant's: "If, in carrying out this process (viz. the proof of a given proposition), we come only on general logical laws and on definitions, then the truth is an analytic one, bearing in mind that we must take acount also of all propositions upon which the admissibility of any of the definitions depends" (*The Foundations of Arithmetic*, transl. Austin, p. 4[c]). Since the associative law of addition, which Frege showed to be tacitly assumed in Leibniz' famous proof of "$2 + 2 = 4$," would hardly have been counted by him as a "general logical law," one might wonder why he decided in favor of the view that arithmetical truths are analytic. The puzzle is solved, however, if we remember that Frege anticipated the reduction of arithmetic to logic which was carried out in detail in Russell and Whitehead's *Principia Mathematica*. That is, if all the axioms of

[18] Cf. above, pp. 12-13.

arithmetic should themselves be provable on the basis of "general logical laws," then Frege would be right in holding all the truths of arithmetic, axioms and theorems, to be analytic in the defined sense. Consider, for example, arithmetic as axiomatized by Peano. The primitive terms "zero," "number," and "successor" need not be interpreted for purposes of formal deduction of theorems from the axioms (supplemented with definitions, explicit or recursive). But the system thus constructed then does not contain any arithmetic *truths* at all, since the axioms as well as the theorems are propositional functions. Given, however, Russell's logicist interpretations of the primitive terms, the axioms become deducible from logical laws,[19] and hence may be said to be analytic in Frege's sense. Notice that Leibniz' proof of "$2 + 2 = 4$," which Frege cited and corrected, does not establish the analyticity of the equation, since the primitive terms used in the definientia, viz. *1* and *successor*, are given no interpretation and hence the derived formulae cannot be said to have a truth-value at all.[20]

Frege's definition of "analytic" explicates well what is meant by saying that the logicist (Russell's) philosophy of arithmetic has refuted Kant and established the analytic nature of arithmetic truths. There is, however, a subtle difficulty here which was touched upon in the discussion of Leibniz and which is generally overlooked. What did Frege mean by "all propositions upon which the admissibility of any of the definitions depends"? Primarily he must have been referring to propositions asserting unique existence, involved in definitions of the form "$y = (\iota x)\phi x$," e. g. the definition of a certain number as the limit of a converging sequence. But he may also have been thinking of the difference between *arbitrary* and *explicative* definitions. After all, any statement could be made to express an analytic truth in his sense if any definition whatever were admissible. Now, the proposition upon which, e. g., the admissibility of Russell's definition of the number one depends is none

[19] I disregard here the difficulty connected with the axiom of infinity—given Russell's definition of "number" and the theory of types, the infinity of the set of natural numbers cannot be proved without assuming the axiom of infinity, and the latter does not seem acceptable as a "logical law"—since it will be discussed later, in Chap. 6; see also Chap. 13, p. 387.

[20] In other words, "analytic" is a semantic, not a syntactic, concept. Compare the distinction made by Carnap in his *Introduction to Semantics* (Cambridge, 1942) between "*L*-true" and "*C*-true."

other than the proposition: $A \in 1 \equiv (\exists x)[(y)(y \in A \equiv y = x)]$, which is intended to express, in the form of a contextual definition, the *correct analysis* of the concept "one." Frege would presumably say that unless this proposition is true (or more accurately, unless the proposition constructed from the above propositional function by universal quantification over the free class-variable A is true), the corresponding definition would be inadmissible. But this proposition could be demonstrated as analytic in the specified sense only by using the corresponding definition as a rule of substitution (thus it would be turned into a substitution-instance of the law of identity), and such a proof would of course be grossly circular. It seems that we are thus confronted with the following choice: either we intend the division analytic-synthetic to *exhaust* the class of true proposition (every true proposition is either analytic or synthetic), in which case we have to admit that the sort of equivalence propositions, just illustrated, which are asserted as the result of logical analysis of a concept, are *synthetic a priori*; or else we shall have to rule that the analytic-synthetic distinction is not applicable to those propositions on whose validity, in Frege's words, the admissibility of the definitions used in a proof of analyticity depends. Since "synthetic" is defined by Kant as the contradictory of "analytic,"[21] we must of course admit that anything of which these terms are significantly predicable is either analytic or synthetic. The question remains, however, of whether the range of significant predicability may not be narrower than the class of propositions. Frege's definition of "analytic" rather suggests that "analytic" is not significantly predicable of analyses (explicative statements) nor of logical laws, since this concept is defined with reference to analyses and logical laws.

[21] The most clear-cut inconsistency (to be distinguished from carelessness) in Kant's use of the terms "analytic" and "synthetic" consists in his so defining "analytic" that all analytic judgments are true, and at the same time defining "synthetic" as the contradictory of "analytic," and yet so *using* "*synthetic*" that *no synthetic judgment* could be self-contradictory. The inconsistency can be resolved by either denying the exhaustiveness of the disjunction "either analytic or synthetic" and replacing it by the disjunction "either analytic or synthetic or self-contradictory," or by substituting in the definition of "analytic" "truth-value" for "truth," such that analytic judgments may be analytically true or analytically false (self-contradictory). Most present-day uses of "analytic" correspond to the former terminology, which is also adopted in this book.

SUMMARY OF PART TWO

THAT all necessary (or "a priori") propositions are analytic, and that accordingly all synthetic propositions must be validated empirically, is a tenet which, at least in part, defines modern empiricism, and thus characterizes this philosophical movement as a reaction against Kantian epistemology—though, on the other hand, it is also continuous with Kant insofar as it inherited from him (1) the distinctions which are necessary for the formulation of its own credo, (2) the general philosophical task of reflecting on "the conditions of the possibility of knowledge," or of "logical reconstruction of knowledge," as this enterprise is nowadays called. However, as the method of analysis has increased in subtlety or sophistication, it has become evident that such a formula, like so many philosophical formulae that are continually used to identify "schools," has a deceptive verbal simplicity which conceals a conceptual muddle.

In the first place, the term "analytic" is sometimes used in the strict sense of demonstrability on the basis of definitions and principles of logic, sometimes in a broader sense which is often expressed as "certifiable as true by reflection upon meanings alone." It is argued in Chapter 5, in connection with an examination of C. I. Lewis' painstaking attempt to prove that all a priori truth is analytic, that the broader sense of "analytic" is not distinguishable at all from the sense of "a priori," so that by this interpretation the empiricist thesis is true but trivial. If, on the other hand, the strict sense is intended, then the latter is confronted by insuperable difficulties: (1) the concept of "logical truth" itself can be clarified only by presupposing an understanding of "entailment," hence of "necessary truth" (this is argued in detail in Chapter 6, in the course of an examination of the standard definitions of "logical truth," especially Carnap's definition); therefore the clarification achieved by substituting "analytic" for "necessary" is illusory. (2) If "analytic" is to be a term of interest to epistemology (not just to formal logic), then, as Lewis has emphasized, the definitions which are to enable such formal demonstration must be qualified

as explicative statements. But characterization of the latter as themselves strictly analytic leads not only to an infinite regress of validation but also to the paradox of analysis. This paradox comes in for detailed discussion in Chapter 10. (3) There are necessary statements, of the "no surface is both blue and red all over at the same time" variety, which could not possibly be shown to be strictly analytic because their predicates designate unanalyzable properties. Much as an empiricist may frown at such claims of absolute unanalyzability (especially if, as is argued in Chapter 9, this turns out to be a psychological concept—"psychologism" again!), he will have to live with them if he accepts the old logical atomism which still underlies Carnap's recent Grundlegung *of both deductive and inductive logic in terms of "state descriptions."*

That the slogan "all necessary truths are analytic" cannot really serve to define, even in part, a unified philosophical school is further evident if we consider that it leaves the important question of "conventionalism" with regard to necessary propositions entirely open. Thus Lewis holds that analytic truth is in no intelligible sense grounded in linguistic conventions, whereas most empiricists would incline toward some form of a linguistic theory of necessary truth. This question is under discussion in Chapter 7. The central issue here is whether the statement that a given proposition is necessary is, if true, itself necessary (an issue occasionally discussed in connection with modal logic), or whether it is a contingent statement about linguistic usage. It is argued that conventionalism is closely allied with the latter position, which however arises from a confusion between "the sentence S, as presently used, expresses a necessary proposition" and "the proposition which happens to be expressed at present by S is necessary." Yet, once the question of conventionalism is dissolved by the exposition of this subtle confusion, there remains a sense of intellectual discomfort about the apparent postulation of propositions as abstract entities. Although the position is taken, in opposition to the thesis of extensionality, that statements about propositions are not translatable into statements about sentences, it is maintained that the conception of propositions as possible objects of belief (and other cognitive attitudes) need involve no mystical platonism.

For a logical empiricist, belief in synthetic a priori knowledge is tantamount to confusion of the a priori and the factual. This

dualism, expressed by the dictum "necessary statements say nothing about reality," is found to be undermined by three central notions developed by empiricists themselves for the purpose of logical reconstruction of scientific knowledge: ostensive definition, implicit definition, and reduction sentence. How the thesis, which is unobjectionable enough, that all meaning derives from ostensive definition of primitive predicates plays havoc with the conception of analytic statements as devoid of existential import, is shown in Chaper 9. The notion of implicit definition, which stems from the axiomatization of formal sciences, has recently been dragged into the logical reconstruction of empirical science as well. Not only are the entities postulated by a physical theory, like electrons, said to be implicitly defined by the axioms of the theory, but necessary statements involving simple descriptive predicates are characterized as somehow "implicitly" definitional. As argued in Chapter 8, this stretching of the notion of definition (and thereby of the notion of analyticity) saves the exhaustiveness of the disjunction "either analytically true or contingently true" only at the cost of destroying its exclusiveness. In this respect the most critical of the three critical notions is perhaps that of reduction sentence; the special problems raised by it, especially the question of whether the analytic-synthetic distinction is best construed, at least in certain contexts, as a distinction of degree, are discussed in Chapter 11.

Chapter 10 contains a discussion of the related notions of analysis and synonymy. It is urged that analyses (the modern counterpart of "real definitions") are about concepts and cannot be construed as empirical statements about synonymous usage of expressions. The notion of identity as applied to concepts (or attributes) leads, however, to the paradox of analysis. As this paradox arises from the requirement of universal substitutivity of names of identical entities, even in nonextensional contexts, a relaxation of this requirement is just the needed cure. But then philosophical analysis will have to operate with a notion of degrees of synonymy (or identity of concepts). This conclusion foreshadows a "gradualism" with respect to the necessary-contingent and analytic-synthetic distinctions as argued for in Chapter 11.

The controversy about phenomenalism seems to have reached in recent years an impasse: it has come to be realized that no sense-statements are analytically entailed by physical statements, but, on

the other hand, there seems to be no other way of specifying what physical terms mean except by pointing to possible sense experience; analogous considerations apply to the language of theoretical "constructs" in relation to the "thing language" (as Carnap calls the prescientific language of everyday life), and the "mentalistic" language of introspective psychology in relation to the language of physiology and behavioristics. It is argued in Chapter 11 that the root of the trouble is the attempt to force statements involving "open" concepts into the analytic-synthetic dichotomy. Instead it should be recognized that an implication may serve to (partially) explicate a concept even though it has a probabilistic character. It is shown that this kind of "gradualism" is already implicit in Carnap's theory of reduction sentences. It is further argued that the concept of "reduction sentence," construed as a kind of interpretative probability implication, enables a better understanding of scientific procedures, especially the procedure of "operational definition" in physics and psychology, than does the contrast analytic-synthetic.

Since the concept of entailment (and the related concepts of self-contradiction and logical incompatibility) is the primary tool by means of which analytic philosophers undertake to analyse concepts—as is amply illustrated in Chapter 13—one might justly complain that analytic philosophy is an obscure enterprise as long as the meaning of "entailment" remains obscure. Chapter 12 addresses itself to the charge that the foregoing analysis is relevant only to the pragmatic meaning of this fundamental logical constant and leaves us in the dark about its semantic meaning. The answer is given that a semantic interpretation of (relative to a given calculus) primitive logical constants—like "(logically) possible" or "not"—, in the sense of analytic definitions in terms of vocabulary intelligible to one who does not already understand them, is impossible. On the other hand, while the related terms "possible," "necessary," "entailment" are conceded to have only pragmatic meaning in some contexts, the thesis of Chapter 7 that "necessary" sometimes stands for an intrinsic property of propositions is reaffirmed, with special reference to those formal entailments in accordance with which we draw deductive inferences.

In Chapters 13 and 14 the "absolute" use of the concepts of entailment and self-contradiction which is characteristic especially of English analytic philosophy is defended against the "formalist" position that these concepts can be significantly used only relative to a language system. It is chiefly by means of intuitive judgments of entailment that the "explicandum" of a given explication is identified. Such insights into meanings lead to formulation of criteria of adequacy, i. e. necessary statements involving the explicandum which must be formally demonstrable on the basis of an adequate explication. Clearly, explication could never get started at all if one withheld assent to the necessity of such statements until an adequate explication is produced. And since the necessary statements in question are of the intuitively evident kind, disagreement about them at the start of an explication is indicative of a difference of interpretation, i. e. that the disputants have different explicanda in mind. In the course of Chapter 13 there is also a resumption of central questions about analysis and necessity which came up in earlier chapters. (1) How can we know the truth-value of a proposition before we know its analysis? (2) What is wrong with the conception of analysis as empirical investigation of linguistic usages? (3) How can we justify a procedure implicitly followed by many "philosophers of ordinary usage," to lay down as criterion of adequacy of an explication of a concept that there should exist instances to which the concept applies? (4) What can we make of the view that a correct analysis must describe the usual method of coming to know that the analyzed concept applies in a given case (criterion of "epistemic adequacy")? Tentative answers to these tricky questions are given within the framework of a conception of philosophical analysis as a mixture of intuition, concept-construction, and formal deduction which should not be confused with empirical sociology or with formal logic or with introspectionist psychology.

The book closes with a critique of "logical relativism," viz. the view that "p entails q" makes sense only if it is elliptical for "p entails q in language-system L." The main point of this critique is that the very choice of an adequate language system as framework for explications presupposes judgments of entailment and incompatibility which cannot, on pain of infinite regress, in turn be relativized to a language system. In this connection the thesis is reaffirmed that modal statements are not translatable into an extensional metalanguage and that in this sense the logical modalities are intrinsic properties of propositions.

CHAPTER 5. *Analytic Truth and A Priori Knowledge*

A. Is "All A Priori Truths Are Analytic" Synthetic?

ONE of the theses that seem to define the rather loosely used word "empiricism," and at any rate the less loosely used expression "logical empiricism," is that all a priori truth is analytic; which thesis is, of course, equivalent to denying that there are synthetic a priori truths. Since the advocates of this thesis assume that the analytic-synthetic distinction is clear, they will surely permit it as a clear and legitimate question whether the universal proposition they assert is itself analytic or synthetic. It is true that according to Russell's theory of types no proposition may make an assertion about itself. Thus, if we observe this ruling, we may not deduce from "all necessary[1] propositions are analytic" that this very proposition is analytic if it is necessary. But no logical empiricist would acquiesce in the admission that at least one necessary proposition of second order (i. e. a proposition about propositions which themselves do not refer to propositions) is synthetic. There are no doubt some members of the school who would advance the thesis as no more than an inductive generalization. Their attitude is that if anyone claims there are synthetic a priori truths, it is for him to produce examples; and they feel confident that no matter which candidate is proposed, they could show (given sufficient time and analytic skill) that it is either analytic or else not a necessary proposition. However, some have made the far stronger assertion that there *can* be no synthetic a priori truths, that to say there are such is as self-contradictory as to say that there are square circles. Schlick, the founder of the Vienna Circle, is unambiguous on this point: "The empiricism which I advocate believes it to be clear that all propositions are, in principle, either synthetic a posteriori or tautological; synthetic propositions a priori seem to it to

[1] "A priori" and "necessary" are here used synonymously.

constitute a *logical impossibility*" (*Gesammelte Aufsätze*, Vienna, 1938, p. 25, translation and italics mine).

Obviously, whether this strong thesis can be maintained depends on what meanings are assigned to the terms "analytic," "synthetic," "a priori," "empirical" (a more customary synonym for Kant's "a posteriori"). Since "analytic-synthetic" and "a priori-empirical" are commonly intended as pairs of contradictories,[2] it will be sufficient to give independent definitions for two out of this quartet of terms, say "analytic" and "a priori," or "synthetic" and "empirical." Some writers define "synthetic" independently of "analytic" in a way which turns it into a synonym for "empirical," and clearly such a usage entails the logical impossibility of synthetic a priori truths. But it is surely absurd to define a traditional term in such a way that, on the basis of the definition, a traditional question formulated by means of that term becomes nonsensical, and then to announce that analysis has revealed the nonsensicalness of the traditional question. It will, therefore, be more profitable to regard "synthetic" as a synonym for "not analytic and not self-contradictory," and to examine whether the independent definitions of "analytic" and "a priori" commonly given warrant Schlick's thesis of the logical impossibility of the synthetic a priori.

The following definition of "a priori," as predicable of propositions, seems to combine clarity with conformity to traditional usage: "A proposition is said to be true *a priori* if its truth can be ascertained by examination of the proposition alone or if it is deducible from propositions whose truth is so ascertained, and by examination of nothing else" (Ambrose and Lazerowitz, *Fundamentals of Symbolic Logic*, p. 17).[3] Now, it is important to note, in connection with the question at issue, that such a definition of "a priori" is silent about the *reason why* any propositions are accepted as true

[2] As pointed out above, p. 46, "synthetic" is the contradictory of "analytic" only if self-contradictory statements are classified as analytic. A good term to use for this inclusive sense of "analytic" is Carnap's "*L*-determinate."

[3] I prefer it to C. I. Lewis' definition "that knowledge whose correctness can be assured without reference to any particular experience of sense" (*An Analysis of Knowledge and Valuation*, p. 35) not just because "assuring the correctness of knowledge" sounds like "assuring the foursidedness of a square," but because "a priori" is here negatively defined in terms of "experience of sense," which term badly needs clarification: is introspection, for example, included in "experience of sense," or the occurrence of moral sentiments? The latter question is particularly pertinent in connection with the problem of a priori knowledge of ethical propositions.

without any appeal to empirical evidence; in ascribing the property "a priori" to a proposition we merely state that it is true regardless of what the empirical facts may be and so could not be refuted by any empirical facts, leaving it an open question what the source of our cognitive satisfaction is. The theory that all a priori truth is analytic must be understood as an answer to just this question—"what is the source of a priori knowledge?"—which claims to avoid the metaphysical mystery in which Kant's answer was wrapped. The reason why such sentences as "all bachelors are unmarried," "$2 + 2 = 4$" *must* be accepted as true as soon as they are understood, says our theory, is that their denial would involve an *inconsistent use of words*. Since, e. g., "some bachelors are married" is synonymous with "some unmarried men are married," one who asserted this could not be using the term "married" consistently. But is there any contradiction in supposing that though a given statement is accepted as true by any rational animal who understands it, regardless of what experience shows to be the case, the reason for such universal acceptance is not the instinctive avoidance of verbal inconsistency? If there is, it still requires to be demonstrated; and if there is not, then the concept of "synthetic a priori truth" is by no means self-contradictory.

The source of the view that the synthetic a priori is a square circle, which is rather popular among contemporary analytic philosophers, is probably a shift of meaning of the term "analytic" from the original restricted sense which Kant gave it to the broader and looser sense "true by virtue of *meanings* alone." To say that S is true by virtue of the meanings of its constituent terms can only mean that an honest denial of S implies (given correct employment of rules of deduction) *misinterpretation* of S. But analyticity in this broad sense does not imply analyticity in the strict sense, viz. reducibility to logical truths. We have seen (Chap. 2, B) that Kant's definition of analytic truth as truth certifiable by the principle of contradiction alone is best clarified as follows: with the help of adequate definitions S can be transformed into a synonymous sentence S' of such a kind that "not-S'" logically entails a contradiction,[4] i. e. a sentence of the form "p and not-p." Now, there is no

[4] If the reader will keep in mind that the concept of analytic truth is here sharply distinguished from the concept of logical truth—contrary to frequent confusions of the two—he will not feel tempted to condemn this definition as circular.

obvious guarantee that if S expresses in a language L an a priori truth in the sense defined (i. e. it would not occur to anybody who understands L to either defend or refute S by an appeal to empirical evidence), then it is analytic in the defined restricted sense.

It is noteworthy that the most elaborate attempt to *prove* the thesis of the analyticity of all a priori truth, viz. C. I. Lewis' in *An Analysis of Knowledge and Valuation*, is vitiated by just such a shift from the wide to the strict sense of "analytic."[5] Let us take a close look at the way Lewis initially formulates the question whether all a priori truth is analytic (the affirmative answer to which question constitutes one of the main theses of his book):

> All analytic statements are, obviously, true *a priori*; whatever is determinable as true by reference exclusively to the meanings of expressions used, is independent of any empirical fact. That the converse relation also holds; that whatever is knowable *a priori*, including the principles of logic and all that logic can certify, is also analytic, is not so obvious. It has, of course, frequently been denied; most notably in the Kantian doctrine which makes *synthetic a priori* truth fundamental for mathematics and for principles of the knowledge of nature [p. 35].

It will strike the attentive reader at once that Lewis' implied definition of "analytic" here corresponds closely to the definition of "a priori" that was approvingly quoted above. Since at the same time he emphasizes that the proposition "all a priori truth is analytic" is, unlike its converse, not self-evident, he ought to provide us with a clearly *different* definition of "a priori." Now there is, indeed, a verbal difference in the definitions of "analytic" and "a priori": an a priori truth is defined as one that is "independent of any empirical fact." Yet, what precisely are we to understand by the statement that p is "independent of any empirical fact"? Does it mean that p could not conceivably be false? But how is this different from saying that analysis of p, supplemented if necessary by application of the apparatus of formal logic, is sufficient for determining p as true? Suppose that somebody who

[5] In spite of the critique of Lewis contained in the following pages, I am greatly indebted to him in another respect, viz. for his lucid attack on "conventionalism" with respect to necessary propositions.

did not understand the meaning of "empirical fact" asked: "Isn't it an empirical fact that all cats are animals? But surely the truth of the statement 'all cats are animals' depends on that fact?" He would, of course, be answered by pointing out that "all cats are animals" does *not* express an empirical fact—which is equivalent to saying that this statement is not empirical. Thus it turns out that, unless the definiens already used for "analytic" is used again for "a priori," the latter term is negatively defined as "not empirical." There would, indeed, be no objection to such a negative definition if a satisfactory positive definition of "empirical" were at hand.

There are three positive definitions of "empirical" (or its synonyms "contingent," "factual")[6] that are widely adopted: (1) a proposition which, if true, *might conceivably* be false, and if false, might conceivably be true; (2) a proposition which is not true (respectively false) in all *possible worlds*; (3) a proposition whose truth-value can be ascertained only by experience. The first definition is particularly reminiscent of Hume, the second of Leibniz (and, via its semanticized twin, of Carnap). Now, it will be remembered from our discussion of Hume that Hume does not use the term "conceivable" at all clearly and univocally, and the same is true to a large measure of those contemporary philosophers who explain their usage of "empirical" by means of the first definition. Specifically, it is not clear whether "the falsehood of p is conceivable" means "not-p is self-consistent" (i. e. no contradiction is logically deducible from not-p) or has a narrower meaning, say "not-p is imaginable." If the former sense is intended, then "empirical" has been defined as the contradictory of "analytic" in the strict sense, the sense which is defined in terms of "logical truth." But in that case "a priori," the contradictory of "empirical," is of course synonymous with "analytic"; it is then a tautology to say that all a priori truth is analytic. And if the second sense is intended, then we get Hume's "psychologistic" criterion of contingency in terms of *imaginability* of alternatives, which we have seen to be

[6] It should be noted that I so use "empirical proposition" that "p is empirical" does not follow from "some of the constituent *concepts* in p are empirical." Thus "all cats are animals" does not express an empirical proposition though the concepts "cat" and "animal" are of course empirical.

unsatisfactory—at least pending a more careful analysis of the relevant meaning of "imaginable" (cf. below, p. 218).

The second definition is equally unilluminating. What does "possible" mean in the expression "possible world"? And the ready answer is: a world described by self-consistent[7] statements. Thus "empirical" is once more defined as the contradictory of "analytic" in the strict sense, and not as the contradictory of "a priori," as it should. The Leibnizian conception of truths of reason that hold in all possible worlds has currently recovered prestige among empiricists, owing to Carnap's translation of this conception into the semantic mode of speech. Instead of speaking, ontologically, of truths holding in all possible worlds, one speaks, semantically, of truths holding in all state descriptions which the primitive names and predicates of the given language system allow to be constructed.[8] But, apart from the consideration that we are here investigating the meaning of "empirical," and cognate terms, as predicated of statements of a *natural language*, not as predicated of statements belonging to a language system, Carnap's semantic reconstruction of the Leibnizian concept likewise fails (one is tempted to say, deliberately fails) to provide a distinction between "empirically true" and "true but not logically demonstrable" (i. e. "non-analytic" in the strict sense of "analytic");[9] hence it is not relevant to our present problem of explicating the "a priori—analytic" distinction, an explication we badly need in order to vindicate Lewis' claim that the thesis of the analyticity of all a priori truth is valid but not trivial.

The third definition of "empirical proposition" is perhaps the

[7] Some may object to the term "self-consistent" on the ground that every statement is consistent with itself. But it seems to be a suitable term to mark out those statements which do not by themselves, without additional premises, entail a contradiction. It is admitted that laws of logic are required as *rules of deduction* for any demonstration of self-inconsistency (cf. p. 75 n.).

[8] For a detailed discussion of Carnap's concept of "*L*-truth," see the following chapter.

[9] The gratuitous character of Carnap's identification of "empirical" truth and "truth which is not logically demonstrable" is noted by Kneale in his penetrating paper "Truths of Logic," *Proceedings of the Aristotelian Society*, new ser. (1946): "But his decision to use the word 'logical' as though it were synonymous with 'a priori'" does not "settle the old dispute about the existence of synthetic a priori truths. If we adopt his usage we merely decree that anyone who wishes to discuss the question with us in future should adopt a new terminology" (p. 230).

most promising, for it turns Lewis' thesis into the thesis that all propositions which *can* be known independently of experience (i. e. a priori propositions) are analytic, which seems to be an interesting, nontautological thing to say. However, when Lewis speaks of analytic "truths," he evidently means (extralinguistic) propositions, not statements, since he emphasizes that analytic truth is independent of linguistic conventions. But then "analytic truth" in the broad sense can only mean truth by virtue of time-independent properties of, and relations between, the concepts (or "*meanings*," in Lewis' terminology) that are constituent of the proposition. If so, is not "analytic truth" synonymous with "a priori proposition" as defined by Ambrose and Lazerowitz? And is not to say that *p* can be known independently of experience synonymous with saying that nothing else needs to be done in order to assure oneself of the truth of *p* except examining the proposition *p* itself? Thus even our third definition of "empirical" fails to bring out a difference between Lewis' broad concept of analyticity and the concept of a priori truth.

A close examination of Lewis' argument confirms our suspicion that his thesis is either a tautology or else false, and that the appearance of significance arises merely from a covert equivocation. When the ambiguous term is used in one sense, the thesis is a tautology, when in the other sense, it is false, and it is because we are unaware of the equivocation that we delude ourselves into supposing the thesis both true and interesting. Specifically, we have a tautology whenever the terms "analytic" and "a priori" are used synonymously and a falsehood whenever "*analytic*" assumes the strict sense of "certifiable by logic plus adequate definitions." The following will serve the purpose of exposing the confusion under discussion: "Every analytic statement is such as can be assured, finally, on grounds which include nothing beyond our accepted definitions and the principles of logic. And statements belonging to logic are themselves analytic; hence capable of being certified from the definitive meanings of the constant terms constituent in them and the syntactic relations of these which they express" (p. 96).

Is capacity of being "assured, finally, on grounds which include nothing beyond our accepted definitions and the principles of logic" a *definition* of "analytic statement" or is it a demonstrable

property of analytic statements? Now, if the former,[10] then it has not been demonstrated that all a priori truth must be analytic in this sense; and if the latter, then "analytic" must be used in that context in the sense of "a priori," and again we have a claim for which proof is wanting. To get directly at one of the main difficulties: what if the descriptive terms by which a given a priori truth is expressed have unanalyzable meanings, such as not to admit of "explicative" definitions? To say that the truth of *S* can, with the help of adequate definitions, be assured on the basis of principles of logic alone can only mean that after elimination of definable descriptive terms we are left with a substitution-instance (*S'*) of a principle of logic—which is to say that all primitive descriptive terms occur *vacuously* in *S'*. It follows that if color predicates, for example, are irreducibly primitive (the sort of predicates one expects to find in Carnapian state descriptions), then such famous a priori truths as "nothing is simultaneously blue and red all over" could not possibly be analytic in the strict sense, for replacement of the descriptive predicates "wholly blue" and "wholly red" with predicate variables does not leave us with a statement form that is true for all admissible substitutions (in other words, the descriptive terms occur *essentially* in the statement.) It is tempting to think that the qualification "all over" marks such truths as strictly analytic after all. Does not "*x* is wholly blue" mean "*x* is blue and has no other color, i. e. is not red, not green, etc. (where *x* is a surface)"? Not so. For surely a man who had seen no color except blue and hence did *not understand the meaning of "red," "green," etc., could* grasp the

[10] At the very end of his chapter 5 Lewis says, indeed, that such a definition of "analytic" would be circular: "Thus, confronting the question, 'What is analytic truth and how do we know it?,' any answer supposedly indicated by taking logical deducibility from definitions as the criterion of the analytic, would be an answer which is circular. Because the acceptability of a definition depends on its being an analytic truth. And the validity of any inference from a definition depends upon the analytic truth of the principles in accordance with which it is drawn" (p. 130). But this quotation merely shows that Lewis himself is anything but clear as to what exactly he means by "analytic." For, since analytic truths are at any rate said to have the mentioned property (though it is not the defining property), Lewis is committed to saying that correct explicative definitions as well as principles of logic "can be assured, finally, on grounds which include nothing beyond our accepted definitions and the principles of logic." And is it not obvious that this method of validation leads into infinite regress if it is applied to explicative definitions and principles of logic themselves? (We shall return to this point on p. 103, below.)

intended meaning of the statement "this entire surface is blue" (we may assume that ostensive definition of "blue" is still possible even though no instances of "non-blue" could be presented, since any two objects might differ in other properties P such that we could say "these two objects are both blue but they differ with respect to P"). Indeed, the statement simply means "every (discriminable) part of this surface is blue." One could express the a priori truth in question as "no minimal discriminable part of a surface is simultaneously blue and red" and thus effectively destroy the appearance of analyticity.[11]

Another way of putting the matter would be to say that a conjunction like "x is blue and red" is not *formally* contradictory or transformable into a formal contradiction by substitution of intuitively adequate definientia; and that it therefore cannot possibly be maintained that the necessary statement "there is no x such that x is blue and red" (where "x" may be taken to range over space-time regions) is a *tautology* in the sense in which the early logical positivists maintained that all necessary truths are tautologies. They did, indeed, notice the difficulty which statements of this kind presented for their theory of necessary truth. In particular, the illustrated incompatibilities between the simple determinates of a common determinable seemed to refute the doctrine of "logical atomism" (as Russell called it), according to which logical relations of entailment and incompatibility are due to the truth-functional complexity of propositions; from which it follows that atomic propositions cannot possibly stand in such logical relations to each other. As reported by Waismann ("Was ist logische Analyse?" *Journal of Unified Science*, 1939-40, sec. 7), Wittgenstein saved his theory by the following reasoning. Let $p = P(a)$, $q = Q(a)$, where "P" and "Q" are incompatible simple predicates. Now the truth-table for "$p.q$" seems to yield the result that this conjunction is not a contradiction, since it is true in one case, viz. if p and q are both true. However, it is nevertheless the case that this conjunction is false for all possible combinations of truth-values of the simple

[11] Reduction to an analytic truth might also be attempted as follows: "x is red" entails "the color of x resembles red more than it resembles green," and "x is green" entails "the color of x resembles green more than it resembles red," which consequences are incompatible. But this argument begs the question. Since the triadic relation "resembling y more than z" is a *primitive* perceptually given relation, the judgment that it is asymmetrical cannot be strictly analytic.

components, for the case TT just is not a *possible* case. If, accordingly, we strike out the first line of the truth-table, we find that the conjunction *is* a formal contradiction, and the entailment "if p, then not-q" *is* a tautology. Moreover, we then obtain the valid equivalence "$p \equiv (p.-q)$," and thus the principle is saved "that in fact the conclusion of a deduction is contained in the premise." But this is surely no successful defense of the theory that all necessary truths are tautologies and all entailments are formal. For the statement "TT is not a possible case" is precisely the modal statement in question, the statement that the atomic propositions p and q are incompatible. And to argue that recognition of this incompatibility is simply the recognition that "if p, then not-q" is a tautology in the sense of being true in all possible cases would be to commit a glaring *petitio*. Similar a priori truths involving what are *prima facie* unanalyzable descriptive terms are: "Whatever has some shape, has some size, and conversely," "whatever is colored, is extended," "if x is warmer than y, then y is not warmer than x," "if x is earlier than y, then y is not earlier than x." But since the problem of unanalyzable predicates will be specifically discussed in a later chapter,[12] we may address ourselves now to other difficulties involved in Lewis' claim.

B. *Are Explicative Definitions and Principles of Logic Analytic?*

In criticizing what he calls the "conventionalist" theory of analytic truth, Lewis emphasizes that the *definitions* which give rise to analytic truth in the epistemologically important sense are not arbitrary stipulations but explications of meanings. This "epistemologically important" sense might be indicated by saying that a statement is analytic in that sense if it is true by virtue of correct analyses of the ordinary meanings of its terms. "All men are mortal" could be turned into an analytic truth by arbitrarily defining "man" as "rational, featherless and mortal biped," but unlike "all cats are animals" it is not analytic in the above sense. Lewis distinguishes three senses of "definition": syntactic statements, like "'U.S.' abbreviates 'the United States of America,'" semantic statements which "relate a symbol to a meaning" (dictionary definitions),

[12] In Chap. 9. See also the remarks on this problem on pp. 153 f., 214 f., 224 f.

and explicative statements which "relate a meaning to a meaning" and thus are not about symbols at all. It is definitions in the latter sense that are the source of analytic truth in the sense in question, and this is the reason why Lewis holds that analytic truth (unlike, of course, its linguistic expression) is not relative to linguistic rules at all. A change of linguistic rules entails a change in the sentence by which an analytic truth is expressed, but it cannot affect the analytic truth itself. Lewis' point is easily clarified (though such clarification may reduce it to a truism). If we change our semantic rules in such a way that "triangle" and "circle" interchange their meanings, then the sentence "all triangles have three corners" ceases to express the analytic truth it formerly expressed; the latter will now be expressed by the sentence "all circles have three corners." The statements whose truth-value is affected by such changes of linguistic rules are empirical statements of the form "*S* expresses, in present usage, an analytic truth." But when I assert the analytic truth itself, "all triangles have three corners," my assertion is not *about* linguistic usage, though it *expresses* a current linguistic usage. Hence no change of linguistic usage can change the truth-value of the statement we intended to make when we said "all triangles have three corners." It appears, then, that Lewis' thesis of the nonlinguistic character of analytic truth is merely a corollary of the truism that the truth-value of a statement *S* does not depend on the existence or nonexistence of a state of affairs *p* unless *S* asserts the existence or nonexistence of *p*.[13]

Now, explicative statements, according to Lewis, differ from the other two kinds of "definitions" not only in not referring to symbols, but also in being true, or false, *a priori*. Thus, if "a man is a rational animal" is meant as an explicative statement, then its proper formulation is "the concept *man* is identical with the concept *rational animal*," not "the symbol 'man' is, as a matter of empirical fact, used synonymously with the symbol 'rational animal,'" or "no English speaking person would apply 'man' to an object which he does not believe to be a rational animal." If all a priori truth is strictly analytic, then such true explicative statements will have

[13] This *reductio ad trivialitatem* should be regarded as a critique of the "conventionalists" (be they strawmen or real) rather than a critique of Lewis. For it is useful to assert a truism while trying to show that a given philosophical thesis, often verbally endorsed, is either meaningless or else trivially obvious.

to be strictly analytic. But does it as much as make sense to say of the statement "the concept *man* is identical with the concept *rational animal*" that it "can be assured on grounds which include nothing beyond our accepted definitions and the principles of logic?" The only way this statement could be validated by logic would be by exhibiting it as a substitution-instance of the law of identity; and in order to do so, one would require as premise a statement which legitimizes the mutual substitution of "the concept *man*" and "the concept *rational animal*." But no other statement could provide a justification of the substitution except the very explicative statement to be validated! Lewis indiscriminately calls the extensional statement "all, and only, men are rational animals" and the intensional statement "the concept *man* is identical with the concept *rational animal*" analytic. The difference might be expressed by saying that the second statement contains the names of the intensions (senses) of the predicates occurring in the first statement.[14] In calling the first statement strictly analytic one just points to the second statement as a sufficient ground of its validity.[15] Hence the latter cannot be characterized as "analytic" in the same sense, and it is only the subtle shift from "a priori" to "analytic" and back again that permits Lewis to characterize explicative statements as likewise analytic. In fact, the attempt to stretch the extension of "analytic" as indicated leads straight to what is known as the *paradox of analysis*. Just as in order to demonstrate the analyticity of "all and only *A*'s are *BC*'s" we need the premise "the intension of '*A*' is identical with the intension of '*BC*,'" so to demonstrate the analyticity of "the concept *being an A* is identical with the concept *being a BC*" we would need "the intension of 'the concept being an *A*' is identical with the intension of 'the concept being a *BC*.'" But if the latter statement were true, then "the concept *being an A* is identical with the concept *being a BC*" would be synonymous with "the concept *being an A* is

[14] The difference between the semantic functions of "men" and "the concept *man*" is like the difference between the semantic functions of "round" and "roundness": as Lewis elsewhere points out, the abstract noun is a name of the intension—"signification," in his own terminology—of its root adjective.

[15] On the relation between intensional statements and necessary extensional statements, cf. above, Chap. 3, A.

identical with the concept *being an A*," hence the explicative statement would, if true, be trivial.[16]

In closely similar manner, the claim that the principles of logic themselves are analytic in the same sense as statements containing descriptive terms essentially (like " all cats are animals ") may be disposed of. Since the nature of " logical truth " will receive detailed examination in the next chapter, we may confine ourselves here to a statement of the chief difficulties. One could, of course, so define " analytic " that there could be no doubt of the analytic character of logical principles: S is analytic if it is either itself a logical principle or is deducible from logical principles with the help of adequate definitions. But on the basis of such a definition, to say that logical principles are analytic would be no more informative than to say that logical principles are logical principles, and Lewis just like the other philosophers who maintained that logical truths are analytic surely intended to make a significant statement, a statement clarifying the nature of logical truth. To illustrate, consider the law of the excluded middle, as formulated in the propositional calculus: $(p)\ (p \vee -p)$.[17] How are we to validate this proposition by reference to logical principles? Indeed, making substitutions in accordance with the definition " $p \vee q =_{df} -(-p.-q)$," we may prove it to be equivalent to " $(p)-(-p.--p)$," which, via the laws of double negation and commutativity of conjunction, is in turn equivalent to the law of contradiction: $(p)-(p.-p)$.[18] Hence we might say that the law of excluded middle has been validated by reference to the

[16] Alonzo Church has correctly pointed out, using Frege's terminology, that all that is required by the truth (not the analytic truth) of the explicative statement (analysis) is the identity of the *denotations* of " the concept *being A* " and " the concept *being BC*," not the identity of their *senses*; hence it is not the case that the truth of an explicative statement entails its triviality (review of M. G. White vs. M. Black, " The Paradox of Analysis," *Journal of Symbolic Logic*, Dec. 1946). A critical comment on this solution of Moore's paradox will be found below, p. 276. The question of whether and how explicative statements admit of an interpretation that would clearly distinguish them from generalizations about linguistic usage is postponed to Chap. 10.

[17] Some logicians formulate the law of the excluded middle in the metalanguage: for every meaningful statement S, either S or not-S is true. But since " it is true that p " expresses the same proposition as " p," the *proposition* expressed by the metalinguistic formulation seems to me to be identical with the proposition expressed by the object-linguistic formula (not " $p \vee -p$," which is a mere schema, but its universal closure).

[18] For typographical reasons, hyphens are used as negation signs instead of the curl of *Principia Mathematica*.

law of contradiction (the supreme postulate of consistency) analogously to the validation of a statement like " all wives are women ": we have shown that the proposition cannot be consistently denied, the only difference being that the definitions required for the demonstration are definitions of logical constants, not of descriptive terms. But, in the first place, in order to show that a denial of the law of excluded middle would be inconsistent with the law of contradiction, we require, in addition to the definition of " $\vee$," another logical principle, the law of double negation: $(p)(p \equiv --p)$. Assuming our primitive notation to contain the connectives " not " (–) and " and " (.), this law is translatable into: $(p)(-(p.-(--p)).-(--p.-p))$, but no further definitional transformation is now possible that would show equivalence to the law of contradiction.[19] What, then, would be the precise meaning of the claim that the falsehood of this law is not " consistently thinkable "? No doubt we can construct an axiomatic system of propositional logic in which the law of double negation occnrs as a theorem, and we might then say that its denial is *systematically inconsistent*, i. e. inconsistent with the axioms of the system. But this is, of course, the kind of inconsistency which the denials of *contingent* statements may likewise suffer from. And what about the axioms of the logical system? It could not be argued, clearly, that they are necessarily true because they cannot be denied without systematic inconsistency, for this would simply mean that their denials are inconsistent with them, which is true of any proposition! And as illustrated above, definitional transformations alone will not in general suffice for revealing self-inconsistency of the denial, but require for that purpose to be supplemented with further axioms of the system;[20] these axioms, accordingly, would still have to be validated in some other way. To add an illustration, translation into the primitive notation of "–" and " . " will transform the axiom of *Principia Mathematica*, $p \vee q \supset q \vee p$, into $-(-(-p.-q).-q.-p)$, and it is not obvious without further axioms (such as commutativity of conjunction) that the latter is an instance of the law of contradiction.

Secondly, if the law of contradiction is the ultimate ground of validity of logical principles themselves, what is its own ground of

[19] It would not be relevant to invoke the *truth-table* test here, since this is no *generally effective* test of logical truth. See the following chapter.

[20] Cf. the critique of Leibniz' criterion of necessary truth, above p. 8.

validity? If it were valid for the same reason as the less "obvious" logical principles which are allegedly "reducible" to it (though, as illustrated, it would be exceedingly difficult to construct a system of logic with the law of contradiction as the only axiom!), then it would be valid because it cannot be denied without being denied; and, surely, even false propositions have that property.

I conclude, then, that it is only the tacit shift to the wide meaning of "analytic," the meaning which coincides with the meaning of "a priori," that permits Lewis to say that statements belonging to logic are likewise analytic. Indeed, reflection upon the meanings of the logical constants occurring in a statement like "if all the members of class K have property P, then any given member of K has P" is sufficient to produce assent to it. But this is to say no more than that it expresses an *a priori* truth. In the same sense a statement in which descriptive terms occur essentially, like "whatever is red, is colored," may be *a priori*, though it is doubtful whether it is strictly analytic.

C. *The Question of Irrefutability*

Our result so far is that the thesis which defines, in part, the school of "logical empiricism" is either untenable or trivial *if* by an "a priori" truth is meant a truth that can be ascertained by analysis of meanings, supplemented by deductive reasoning if necessary, alone. But it may be replied that what is here taken as the meaning of "a priori" is really intended, by those philosophers, as the definition of "analytic," and that the fundamental meaning of "a priori" is something else: *irrefutability by experience*. That the *reason* for the empirical irrefutability of a statement, it might be countered, is that to deny it is to change the meanings of some of its constituent terms, is by no means trivially obvious. If it were, Kant would have found it obvious, too, and would not have contrived his metaphysics of Reason as an explanation of such a priori truth. The character of the Kantian sort of explanation, which may be historically contrasted with the positivist sort of explanation, might be brought out by a simile. Suppose we found, as a "brute fact," that whatever we, or any other human being, saw looked blue. One obvious explanation of this fact of experience might be that everything that is visible to any being *is* blue (and the

explanation is not tautological either, since "x appears blue" is not synonymous with "x is blue"). On the other hand, we might discover that our eyes and nervous system are so constituted that only light waves whose wavelength corresponds to this color can produce a color sensation. This sounds like the Kantian story: discover the limits of "possible experience" by discovering the "subjective conditions" of experience. Now, the point here is that the statement "everything that is visible is blue" which, *ex hypothesi*, no experience could refute, is surely not true by virtue of its *meaning*. In just this way, Kant thought that no experience could refute the axiom "the straight line is the shortest distance between two points," but that this could not be inferred from the meanings of subject and predicate, but only from the nature of our spatial intuition. That his explanation, once divested of the metaphorical elements that lend pseudo-intelligibility to metaphysical theories—the "forms" of Reason which the "stuff" of experience must conform to—reduces to a tautology, since "our mode of spatial intuition" can hardly mean anything but "finding such propositions as the Euclidean axioms evident," need not detain us in this context.

Yet it is idle to compare alternative explanations of the empirical irrefutability of such propositions as the propositions of arithmetic or the propositions of logic, if we are not clear about the meaning of "empirically irrefutable." It is *impossible* to make observations that would disconfirm "$2 + 2 = 4$" (assuming that this is meant as a statement of pure arithmetic, involving the notion of addition which Russell has explicated in terms of the notion of logical addition of classes, not as a statement about results of physical operations of addition). Yes, but what kind of impossibility is meant? Since many empirical statements are such that it is *practically* impossible to disconfirm them—the "there are mountains on the other side of the moon" variety—it is evidently *theoretical* impossibility of disconfirmation that is in question. Does "p is an a priori truth," then, mean "it is logically impossible (i. e. self-contradictory) to suppose that some experience showed p to be false"? This condition will no doubt be satisfied if "not-p" is itself self-contradictory. But if it is not at least logically possible that it should be satisfied if "not-p" is self-consistent, then the suggested definition will, contrary to its claim, have failed to distinguish the meaning of "a priori" from the meaning of "analytic"

—in which case it will be absurd to speak of an "explanation" of apriority in terms of analyticity.

Now, *prima facie* there are propositions of such a kind that the supposition of their empirical disconfirmation seems self-contradictory though they themselves can be denied without self-contradiction. But it will turn out that (a) it is at best the supposition of their complete (conclusive) disconfirmation which is self-contradictory, not the supposition of their *partial* disconfirmation; (b) that such propositions can easily be imagined to be false, and hence could not count as a priori truths anyway. The point can easily be established in terms of two, formally similar, illustrative propositions: the principle of causality in the unsophisticated formulation "every event has a cause," which Kant and his followers held to be both synthetic and a priori, and "every class of empirical objects which has at least one member has at least two members." To refute the proposition that every event has a cause would be to establish the proposition that at least one event has no cause. This latter proposition has the form: $(\exists x)(y)(-yRx)$, where the variables "x" and "y" range over events. Clearly, if the range of these variables is unlimited, then complete verification of this proposition (and thus complete disconfirmation of its negation) would involve exhaustion of an infinite class, which most philosophers (with the notable exception of Russell) would regard as logically impossible. *Mutatis mutandis*, the same may be said of the second of our illustrative propositions. To refute this proposition would mean to establish that there is a class K which has exactly one member.[21] Here again, an unlimited class of elements would have to be exhausted before the proposition could be pronounced as definitely refuted. Thus it appears that the claim of irrefutability here is equivalent to the claim that unrestrictedly universal propositions are unverifiable. But few philosophers would deny that such universal propositions are at least confirmable; hence consistency requires them to concede that evidence is conceivable which would partially disconfirm such propositions as the above in the sense of diminishing their probability. And as for (b): it surely is conveivable that there is not, never has been, and never

[21] It is assumed that the K's are defined by predicates that leave it a *logically open* question whether the class has more than one member. Such classes as the class of tallest men born in 1915 are therefore excluded from the range of "K."

will be, another individual that exactly resembles, in appearance, Winston Churchill; so the fact that it cannot be strictly disproved that the universe contains such a double of Churchill's is no proof at all that the existence of such a double is an a priori truth. Nor do we need to repeat Hume's arguments against the logical necessity of the principle of causality.

It is, indeed, impossible to find a meaning of the expression "theoretically irrefutable by experience" which at once differs from the meaning of "not capable of being self-consistently denied" (i. e. strictly analytic) and effectively serves to distinguish a priori lrom empirical truth. It is sometimes held that the mark of an a priori truth is that we adjust our linguistic rules in such a way that it could be maintained no matter what facts may turn up. Take, for example, the a priori truth that every tone is characterized by exactly oue pitch. Suppose we try to refute it by sounding a chord. Then the defender of the proposition will, of course, say that a chord is not properly describable as a tone. Or suppose we try to refute the proposition "no particle can occupy more than one place at a time" by pointing out that this tennis ball, after all, occupies a spherical volume which is an aggregate of many "places." Then we would be told either that "place" was meant in the sense of "the total space occupied by the particle," or that "particle" was meant in the sense of "what occupies no more than one place at a time" ("point-particle," i. e. its instantaneous position can be described by a *unique* set of coordinates). This linguistic theory ol the nature of a priori truth is nicely argued by Ernest Nagel among others. In the article "Logic without Ontology"[22] he applies the theory to the Aristotelian version of the principle of contradiction as an "ontological" truth: no subject can both have and not have an attribute at the same time and in the same respect. Suppose someone claimed as an exception to the principle the fact that the same penny may at the same time be both circular and ellipitical (and thus noncircular) relative to different angles of vision.[23] He would, of course, be told that the difference of per-

[22] In *Naturalism and the Human Spirit*, ed. H. Krikorian; reprinted in Feigl and Sellars, *Readings in Philosophical Analysis*.

[23] If a phenomenalist analysis of physical properties as dispositions to produce sense-data of definite kinds under definite conditions of perception is accepted, then the distinction between "attribute" and "respect" is unnecessary. If "x is round" means

spectives just constitutes a violation of the requirement of "sameness of respect." But what should we tell him if he asked why the sameness of the degree of illumination of the penny's surface is not, in this context, identified as sameness of respect? As Nagel points out, the proper reply would be that the nature of the "attribute" in question determines our definition of "same respect," i. e. once an attribute (like shape, or color) has been selected, the "respect" (normal perspective, or standard illumination) must be so chosen that the principle remains valid.

But, unfortunately for the linguistic theory, this method of "saving" propositions in the face of apparent exceptions by suitable linguistic conventions is equally applicable to propositions ordinarily regarded as empirical. Consider once again the example used by Poincaré to illustrate the transformation of generalizations from experiment into "conventions": under standard pressure, all phosphorus melts at 44° C. The orthodox theory, which may be called the "either-or" theory of philosophical semantics (not to be confused with the two-valued Aristotelian logic which Korzybski and his followers decry because they do not understand it!), is simple enough: either "phosphorus" is defined in terms of this melting-point, in which case we have an a priori truth which "says nothing about reality"; or else "phosphorus" is not so defined, in which case we have an empirical generalization which is clearly refutable by contrary instances. This theory overlooks two points, the first of which will occupy us at length in Chapter 11 of this book. (1) Such concepts of natural kinds as "phosphorus" are *open* concepts in the sense that no fixed rules of application have been laid down that would cover all possible situations in which the question whether the concept applies might arise, and that would allow us to make a sharp distinction between defining and accidental properties of members of the kind. (2) Such generalizations are usually *incompletely* stated, as is evident from the accompanying safety clause "other things being equal," and it therefore is not clear what would count as a "contrary instance."

(1) It is conceivable, indeed, that a scientist would announce,

"x would appear round if viewed from a normal angle," then it does not make sense to ask whether x *is* also round relatively to non-normal perspectives. But this matter is irrelevant to the point Nagel wished to illustrate by the Aristotelian principle of "ontology."

after having discovered that each element capable of existing in different states of aggregation is characterized by a unique melting-point, that such elements are *defined* by their melting-points—in just the way in which many modern chemists would say that the elements are defined by their atomic weight, or perhaps by their atomic number. But it can be shown that he does not mean "definition," in such contexts, in the sense which underlies the dichotomy "true by definition–true by empirical fact." For suppose that specimens turned up frequently which resembled phosphorus in all respects that were used to identify substances as "phosphorus" before the melting-point was discovered, but which melted at a differeut temperature. Would the scientist refuse to classify these substances as "phosphorus" on the authority of his "definition"? Clearly not. Since his selection of property P_1 out of a group of normally correlated properties $P_1 \cdots P_n$ as "definitory" purports mainly to identify a reliable sign of the other members of the group (such that to classify on the basis of P_1 is to make implicitly a number of predictions), such a procedure would defeat the very purpose of "definition" of natural kinds. Indeed, what he would be more likely to do is to *change the definition in the light of experience*, and thus save the hypothesis that the specimens are phosphorus. The advocates of the either-or theory will say, somewhat cynically, that this is well known but has no tendency to show that the distinction between definitional and empirical truth cannot be maintained in such contexts. If to say that the definition of "phosphorus" in terms of P_1 is *useful* is to say that P_1 is a reliable sign of $P_2 \cdots P_n$, then this statement is empirical; and the fact that after finding it to be false we abandon the definition merely illustrates that the advisability of adopting a given linguistic rule may depend on empirical facts. Yet, what is the criterion for deciding whether a given statement which is part of a scientific theory is a definition or a synthetic statement? What is the criterion for deciding whether a modification of a scientific theory which was occasioned by new observations is a change of definition or an abandonment of an empirical hypothesis? Certainly, the scientists themselves speak of redefinition in the light of new experience, but the question is what they mean by "definition." When it is said that a definition is a statement to the effect that two expressions are synonymous (and thus clearly a metalinguistic statement, to be

distinguished from object-linguistic statements in which the defined expression is *used*, not *mentioned*), one must have one or the other of the following meanings of "definition" in mind: (a) definition as introduction of an abbreviating synonym, (b) definition as explicit statement of what is meant by an antecedently used term (generalization about actual usage, though in stating the generalization as a "definition" one may also *prescribe* future conformity to the described usage, especially if actual usage is to a degree vague). But surely the scientist's statement "phosphorus is defined as the element with melting point M" cannot be adequately translated into "'phosphorus' shall henceforth be used as an abbreviation for 'element with melting-point M'" (the way a mathematician might rule "'x^3' is to be used as abbreviation for '$x.x.x$'"), nor can it be adequately translated into "'phosphorus' as used by us scientists has always meant 'element with melting-point M'": if it had always meant that, why bother to discover the melting-point by experiment? It remains the case, therefore, that this usage of "definition" does not fit into the conception of definition as a metalinguistic statement (whether declarative or not) which is about symbols in a sense in which this is incompatible with being about the objects symbolized. No chemist would admit that in "defining" oxygen as the element with atomic weight 16 he has made the claim that the existence of a sample of oxygen with a different atomic weight (isotopes!) is as logically impossible as the existence of a square circle. What he intended to do in laying down the "definition," whether or not he was able to formulate his intention clearly, was to indicate which member of a group of normally correlated properties is the most reliable indicator of the other members.

The concept of "correlation" which is intended when one says that a name of a natural kind stands for a correlation of properties requires, of course, analysis. To say that the set $P_1 \cdots P_n$ are thus correlated does not mean that any instance exhibiting one will exhibit all nor that it will exhibit a subclass of them; nor does it even mean that any instance exhibiting n-1 members of the set will exhibit the nth member, for in that case there could not be distinct natural kinds that differ in just one property. The relations between the members of a correlation are only probability implications, and the name of a natural kind crystallizes, as it were, a network of probability implications. In arguing "x has P_1, therefore x is a member of K" we are telescoping a whole set of probable inferences to the presence of the other members of the correlation which "K" stands for. If so, it cannot be supposed that some member of the correlation, no matter how complex, could serve as a property which is *logically equivalent* to the property of being a member of K: the inference to class membership is a probable inference to *other* members of the correlation which "K" names. This is the reason why "definition" of a natural kind is not "definition" in a sense usually recognized by conventional logicians.

In his early book, *Mind and the World Order*, C. I. Lewis developed what he himself called a "pragmatic" conception of a priori truth, according to which it is the mark of an a priori proposition "all A are B" that if we find, empirically, that the concept B does not apply to x, then we are determined to retract A as likewise inapplicable to x. What Lewis unfortunately overlooked is that this pragmatic criterion does not allow us to distinguish strictly analytic from empirical truths. It is obvious that high confidence in the truth of "all A are B" will manifest itself in just such behavior regardless of whether the source of this confidence is empirical or logical evidence.[24] What is here suggested is that when a scientist announces a *definition*, suggested by empirical discoveries, of a natural kind, such as "oxygen is defined as the element whose atomic weight is 16," he simply expresses the sort of confidence which Lewis took to be the touchstone of a special kind of truth; that he is not making a terminological proposal of the kind "let us use the term 'oxygen' to refer to the element whose atomic weight equals 16" but rather announces "atomic weight 16 is the most reliable indicator of the members of the correlation of macroscopic and microscopic, qualitative and quantitative properties which constitutes the natural kind called 'oxygen'; in general, classification of chemical elements on the basis of their atomic weights has, as demonstrated by Mendelejeff's periodic law, more predictive fertility than classification on the basis of other members of such correlations." The temptation is, indeed, great to ask: just how are we to *delimit* the correlation of properties which, as you say, a common noun like "oxygen" stands

[24] Cf. on this point my *The A Priori in Physical Theory* (New York, King's Crown Press, 1946), Pt. I.

for? Surely, if today chemists discovered a new property, P_{n+1}, of oxygen, you would not say that "oxygen" connoted that property, along with $P_1 \cdots P_n$, before the discovery was made? But the question is inappropriate because it rests on the traditional conception of *connotation* of class terms as giving rise to a sharp division of properties into essential and accidental; it presupposes, that is, that a name of a natural kind has at any given time a fixed definition, though one fixed definition gives way to another under the pressure of empirical discoveries. The fact is, however, that scientists operate with *open* concepts of natural kinds. If "oxygen" were stipulated to be synonymous with the correlation $P_1 \cdots P_n$, then the scientist would be committed to classify an instance of the subcorrelation $P_1 \cdots P_{n-1}$ which lacked P_n as non-oxygen, while actually he may find it more convenient to classify such a specimen as a new species of oxygen in view of its strong similarity to instances of the correlation $P_1 \cdots P_n$. It is only blindness to the openness of such class concepts that can lead a philosopher of science to assimilate a statement like "this sample of oxygen does not have atomic weight 16" to a statement like "this square does not have four sides."[25]

Now to point (2), that commonly laws are incompletely expressed, through insertion of the "caeteris paribus" clause, and therefore are not strictly refutable.[26] The scientist, knowing that he does not know for certain that the conditions $C_1 \cdots C_n$ are *all* the conditions necessary for an occurrence of effect E (i. e. constitute a strictly sufficient condition for E), even though E has so far been observed to occur whenever this set of conditions was present, cautiously says "provided all the other necessary conditions are present, then, if . . ., then E." Thus a physicist experimenting with wires in order to find out how the length of a wire varies with the stress it is subjected to discovers direct proportionality of stress to strain. What he has strictly discovered, of course, is only that such a functional dependence holds within the experimentally explored range of the variables. So he says cautiously "under similar relevant

[25] The subtle problem of the "openness" of scientific concepts will be discussed in detail in Chap. 11.

[26] On this point, as well as on the just discussed openness of inductive concepts, see the penetrating remarks by G. H. von Wright, *Treatise on Induction and Probability*, chap. 6, secs. 2, 3.

circumstances this dependence always holds," and he may not know at the time exactly what all the relevant circumstances are. The subsequent discovery that above a certain limit of stress (which varies from material to material and which is called "the elastic limit") the law does not hold, does not, then, compel him to abandon the proposition he asserted. In a case like this, one commonly speaks of "modifying" or "restricting" the law to special conditions. But if by the "law" is meant the definite proposition devoid of safety clause, then this would be properly described as refutation, not modification, and if what is meant is the indefinite proposition provided with the safety clause, then "precisification" would be a more suitable term: it is specified just which the "relevant" circumstances are.

The trouble with the claim that refutability is the mark of empirical laws is, then, this: if by the "law" we understand the proposition which the scientist usually means to assert (whether or not he says exactly what he means), we find that the law has a characteristic indefiniteness by virtue of which it is safe from strict refutation; and if by the "law" we mean a strictly refutable proposition, we might find people, laymen as well as scientists, denying that they ever intend to assert laws in that sense. The indefinite formulation, of course, corresponds to the assertion of a probability implication: uncertainty as to what circumstances are relevant reflects itself in uncertainty of the inference from the specified conditions to the effect. It is often said nowadays that we must abandon the "quest for certainty" in science and be content with the assertion of just such probability implications. But if what is called "law" is a probability implication, then either of two things must be meant by a "law": (a) a statistical generalization, i. e. a statement of the form "in x per cent of the cases, condition C is attended by effect E,"[27] or (b) a statement of the form "relative to the total available evidence, the degree of *confirmation* of the statement 'whenever C, then E' is p." The difference between these two interpretations should be obvious: according to (a), there are *known cases* of C which are not attended by E, while according to (b) it just is not known for certain that there are not any excep-

[27] This type of formulation is aimed at one particular form of law, viz. causal laws, or laws of succession. For classificatory laws ("laws of coexistence") the formulation would be "x per cent of the A's are B's."

tional cases. Now, notoriously, we have no definite rules telling us just what evidence would clearly refute a statistical generalization which refers to an infinite class (this is one of the reasons why some people reject the frequency theory of probability: it makes, they say, probability statements undecidable). And according to interpretation (b), a law is itself a statement of "inductive probability" (probability attributed to hypotheses relative to inductive evidence) and as such is analytic if true.[28]

Suffice it to conclude that the term "law" is not sufficiently unambiguous (even within the context of usage indicated by the appendix ". . . of nature") to allow us to pronounce with confidence that laws, unlike a priori truths, are *refutable* propositions. However, even if the difficulty created by the "caeteris paribus" clause could be taken care of, there wonld remain the unanswerable argument of the "conventionalists" (and some of the analytically minded "pragmatists") concerning the illusoriness of "crucial experiments," i. e. the view that experiments can always, theoretically, be designed that would bring to light brute facts clearly deciding against one or the other of two incompatible theories. That in order to deduce directly testable consequences from a theory to be tested we need to supplement it with other theories used as premises—e. g. the Einstein Red-Shift, which is one of the important empirical consequences of the general theory of relativity, cannot be deduced from the latter without the supplementary assumption of a law of the gravitational field, the law correlating wave length of light with color etc.—and that therefore the theory in question could always be saved in case of a negative result by suitably modifying those supplementary theories, has often been pointed out and has now the status of a truism of scientific methodology. But it seems to me to be a truism which is absolutely fatal to the claim that a priori truths are sharply distinguishable from empirical truths by the criterion of empirical irrefutability.[29]

[28] It is here assumed without argument that a satisfactory frequency interpretation of inductive probability is impossible.

[29] The above discussion of the conventionalist claim that experience by itself, isolated from a framework of conventions, could never strictly refute a law has been kept—perhaps unduly—brief, because the matter has been dealt with extensively by several writers, including myself in *The A Priori in Physical Theory*.

D. *Are Necessary Propositions Necessarily Necessary?*

If all of this is correct, then we come to the conclusion that there is no way of improving on the definition of "a priori truth" which constituted our starting point, viz. a true statement whose *truth* is ascertainable by reflecting on its meaning alone, or by logical deduction from statements of this sort. In which case, as pointed out, it will be difficult to escape from the conclusion that the thesis "all a priori truth is analytic" is either a tautology or else false. But let us make one more effort to see clearly what, on the above definition, we are saying about a statement in calling it "true a priori." Do we mean that any rational person could be brought to assent to the statement by just carefully explaining to him what the statement means (by definition of "rational," of course, a rational person would be able to follow a formal demonstration)? If so, then "p is a priori" would be a prediction[30] of psychological reactions, and we would only be warranted in saying, with regard to any given p, "the evidence makes it probable that p is a priori." Now, that "p is a priori" should itself be an empirical statement would be an innocent consequence if it were meant in the sense of "this sentence is presently used to express an a priori proposition." But this very statement about usage suggests that "a priori" is directly predicable of *propositions*, though derivatively of sentences that express a priori propositions. And would any philosopher be willing to admit that once we know which proposition p is expressed by a given sentence, it is still a *question of empirical fact* whether p is a priori? Should we not say, rather, that necessity is an *intrinsic* property of a proposition in the sense that it would make no sense to suppose that a necessary proposition *might not have been* necessary?

The question whether "it is necessary that p" is, if true, itself a necessary proposition is of fundamental importance for the problem of explicating the concept of necessary truth, since it is likely that any philosopher who answers it affirmatively will adopt the necessity of the necessity of p as a *criterion of adequacy* for proposed explications of necessary truth. He will, in other words, reject any explica-

[30] We could hardly define "S expresses a necessary truth" to mean "understanding the meaning of S *entails* believing S to be true," which would be grossly circular. Thus we are justified in speaking of a prediction of belief on the basis of understanding.

tion which entails the contingency of such modal propositions as failing to explicate the explicandum he has in mind. The same holds, of course, for the concept of logical truth: since all logical truths are necessary truths (whether or not the converse of this proposition be true also), any criterion of adequacy for explications of "necessary truth" is at the same time a criterion of adequacy for explications of "logical truth." This question cannot be decided by formal reasoning within an uninterpreted system of modal logic,[31] containing the usual explicit definition of "necessary" in terms of "possible": p is necessary = not-p is not possible. Indeed, an uninterpreted system of modal logic can be constructed without even raising the question of the necessity of the necessity of p; thus there is no postulate or theorem in Lewis' system S_2 that bears on the question, nor is the qnestion informally discussed in the metalanguage. In Appendix II to Lewis and Langford's *Symbolic Logic* (New York and London, 1932) it is pointed out that Lewis' system of strict implication "leaves undetermined certain properties of the modal functions, $\Diamond p$, $\sim \Diamond p$, $\Diamond \sim p$, and $\sim \Diamond \sim p$." Accordingly "$Np \dashv NNp$," as well as "$Np \supset NNp$" ($N \cdots$ = it is necessary that $\cdots$), is both independent of and consistent with the axioms of the system, and whether an axiom of modal iteration, e. g. "what is possibly possible, is possible" (which can be shown to be equivalent to "what is necessary, is necessarily necessary") should be adopted must be decided by extrasystematic considerations based on *interpretation* of the modal functions. Now, let us refer to the thesis that necessary propositions are necessarily necessary henceforth as the "*NN* thesis." What appears to be the strongest argument in favor of the *NN* thesis is based on the semantic assumption that "necessary" as predicated of propositions is a *time-independent* predicate,[32] where a "time-independent" predicate is defined as a predicate P such that sentences of the form "x is P at time t" are meaningless. The argument runs as follows.

Anybody who maintained that the proposition "it is necessary that every father have at least one child" is itself contingent, could

[31] By calling the system "uninterpreted" I mean that its metalanguage contains no semantic rules for the modal operators, not that it contains no semantic rules at all.

[32] The term "time-independent predicate" is borrowed from Carnap, who uses it in "Truth and Confirmation" (Feigl and Sellars, *Readings in Philosophical Analysis*) in order to show the difference between the concepts of truth and confirmation.

only mean that the *sentence* "every father has at least one child," which is in fact used to express a necessary proposition, might have been used to express a contingent proposition (e. g. "father" might have been used in the sense in which "man" is used). Indeed, a statement of the form "S expresses a necessary proposition" is incomplete, requiring expansion into "S expresses, in present usage, a necessary proposition," and once so expanded shows itself as a contingent, indeed historical, statement about verbal usage. But while the predicate "expresses a necessary proposition" is clearly time-dependent, "necessary" as predicated of propositions is just as clearly time-independent. This can be shown convincingly if we consider statements of the form "p entails q," which are reducible to the form "p is necessary" where p is a conditional proposition[33] (thus "that somebody is a father entails that somebody has at least one child" is reducible to "it is necessary that if somebody is a father, then somebody has at least one child"). Suppose a logic teacher—engaged in the demonstration that premises of the forms "all A are B" and "some A are not C" entail that some B are not C, while the entailment does not hold if the universal premise is replaced by its converse "all B are A"—were asked by a befuddled yet critical student: "You have shown that the conclusion is entailed by the first pair of premises but not by the second pair *at the present time*. How do you propose to prove that these logical relations will *always* hold?" Surely this student will have to be told that he has not understood what is meant by "entailment," that in terms of the intended meaning of "entailment" his question does not make sense; that he might just as well have asked how we know that the square root of 9 was equal

[33] It should be noted that the variables "p" and "q" are here meant as propositional, not sentential variables. This is to say that the admissible substituends for "p" and "q" in "p entails q" are names of propositions, which can be formed in two ways: either by prefixing "that" to the sentence expressing the proposition (sentences, be it noted, are not *names* of propositions any more than predicates of their intensions), or by constructing participial expressions corresponding to statements ("somebody *being* a father" corresponding to "somebody is a father"). The claim, which may be traced back to Carnap's *Logical Syntax of Language*, that entailment statements are metalinguistic statements containing names of sentences, is as untenable, and for similar reasons, as the claim that predications of truth, in ordinary language, are metalinguistic statements about sentences; cf. my "Note on the Semantic and the Absolute Concept of Truth," *Philosophical Studies* (Jan. 1952), and "Propositions, Sentences and the Semantic Definition of Truth," *Theoria, 20* (1954).

to 3 before the symbols " 3 " and " 9 " were invented by mankind. Thus it must be concluded that it is inconceivable that an entailment which in fact holds from p to q should fail to hold between the same propositions at some other time, simply because it does not make sense to say that an entailment holds *at some time*.

But this argument for the *NN* thesis assumes that the only ground on which the *NN* thesis could be rejected is the interpretation of " p is necessary," like " S expresses a necessary proposition," as a *historical* statement; once this assumption is granted, the *NN* thesis is easily established, since it is easy to show that modal statements are not historical statements. If this assumption is challenged, then some other argument is needed. Such an independent argument might be constructed on the following premise: if and only if S expresses a *contingent* proposition p, then it is possible that two people who both take S to express p nevertheless disagree about the truth-value of S. Let " p " be of the form " p entails q " (i. e. " it is necessary that if p, then q "). The question before us then reduces to the question whether two people who are in disagreement as to whether proposition p entails proposition q may nevertheless be interpreting the sentence " p entails q " in the same way. And since it is clearly possible that they should agree in their interpretation of " entails " and yet disagree as to whether p entails q, this is to ask whether the disputants could put the identical interpretation on the sentences " p " and " q." Some analysts would deny this possibility and hence conclude that true entailment statements are themselves necessary; thus Schlick said that correct interpretation and verification coincide in the case of analytic statements. But it seems to me obvious that they are wrong. Consider an entailment between fairly complex propositions, e. g. propositions of the forms " if p, then (if q, then r or not-s) " and " if q, then (if s and not-r, then not-p)." They in fact entail one another, but a person not trained in formal logic may have difficulty seeing this and hence might conceivably dispute the entailment. Yet, if he explicitly agrees to the truth-table definitions of the connectives and moreover shows himself familiar with the relevant syntactic rules, it could hardly be denied that the two sentences express for him the same propositions as for the logician who recognizes their logical equivalence.

But then the only possible source of the disagreement is that

one of the disputants is either unaware of some relevant rule of deduction or else employs an invalid rule of deduction or else makes a mistake in applying the relevant rules of deduction; for short, let us say that he commits a deductive error. Should we conclude, now, that the *NN* thesis is false, since the criterion of necessary propositions—viz. that if the proposition p expressed by S is necessary, then anybody who honestly denied S would not interpret S to mean p—does not seem to be satisfied by entailment propositions? Such a conclusion would be most unwise, since the same argument would prove that not even *first-order* propositions (i. e. propositions which are not about propositions) of appreciable complexity are necessary. Deductive mistakes might lead one, for example, to deny an arithmetical equation which expresses a necessary proposition.[34] And if the best reason one can adduce against *NNp* (where *Np* is equivalent to an entailment proposition) is at the same time an argument against *Np*, then the thesis that *if Np, then NNp*, has in fact been supported rather than undermined.

On the other hand, a seductive argument *against* the *NN* thesis is that to establish a proposition as necessary we must, in many cases, perform calculatory or deductive operations which are subject to error just like the processes of interpreting sense data that enter into the verification of empirical propositions about the physical world. Consider, e. g., the truth-table test for determining whether a given formula expresses a tautology. If after performing the test we say confidently that the proposition expressed by the formula is a tautology, it is because we confidently assume that no mistake was made in the calculations of truth-values. Since there is a finite probability of error in the calculation (proportional, roughly, to the complexity of the formula), we ought to say critically " on the evidence of the calculation test, it is highly probable that the proposition expressed by the formula is a tautology "; and since it does not make sense to ascribe probability to a necessary proposi-

[34] If by " understanding " an equation one means " knowing what the constituent symbols mean and understanding the meaning of their arrangement *and having a perfect grasp of the relevant rules of calculation*," then it would presumably follow from this very definition of " *understanding* " that it is impossible to understand an equation and still be uncertain about its validity. But such a definition would be an excellent example of a question-begging definition.

tion,[35] so the argument might conclude, "p is a tautology" is not itself a necessary proposition.

Before exposing the subtle error of this argument, I wish to reduce it to absurdity by showing that if it were valid it would prove that there are no tautologies at all, not even in the realm of first-order propositions. Let us ask ourselves by what sort of evidence we could establish the *truth* (not necessity) of a proposition of the form "p or not-p" (to take the simplest case of a tautology), say "p_1 or not-p_1," where p_1 is an empirical proposition. The naive method would consist in verifying either p_1, or not-p_1, and then inferring, with the help of the principle "p entails (p or q)," the truth of "either p_1 or not-p_1." Clearly, in using this method we establish the disjunction as an *empirical* truth only. But if we are not naive, i. e. if we recognize that the truth of this compound proposition is independent of the truth-value of its atomic component, then we shall adduce evidence for the truth of "p_1 or not-p_1" which is at the same time evidence for the *necessary* truth of this proposition: the evidence of the truth-table. That is, we assert the truth of "p_1 or not-p_1" on the ground that any proposition of the same form is true, i. e. on the ground that it is *necessarily* true. It follows that if the evidence of the truth-table be regarded as empirical evidence which only lends probability to the propositions it is evidence for, then not only "'p_1 or not-p_1' is a tautology" but likewise "p_1 or not-p_1" itself is an empirical proposition. (Note that the truth-table verdict is the only evidence on which "p_1 or not-p_1" can be asserted in case the truth-value of "p_1" is unknown.) Analogous considerations apply if the test by which a formula is established as a tautology is the deduction test.[36]

[35] Perhaps this premise is needlessly strong. One might hold that the propositions of inductive logic, of the form "$c(h/e) = p$," hold for necessary as well as for contingent values of "h." But the inductive irrelevance of empirical facts to necessary propositions might be expressed by the axiom that for any e, $c(h/e) = 1$ if h is a necessary proposition. And this premise would serve the above argument just as well.

[36] The argument against the *NN* thesis which has just been criticized may be found in Reichenbach's *Elements of Symbolic Logic*, § 34. Reichenbach's brief discussion of the question whether "p is a tautology" is itself a tautology (if true) is, moreover, confused, since no distinction is made between the statement that a given formula expresses a tautology and the statement that the proposition expressed by a given formula is a tautology. The former type of statement is obviously empirical, but this has no tendency to prove that the latter type of statement is empirical. In general, the failure to distinguish between "S expresses a necessary proposition" and "the proposition p (which happens to be expressed by S) is necessary" is widespread, especially among advocates of the linguistic theory of logical necessity (cf. Chap. 7). It is responsible, for example, for Strawson's, in my opinion invalid, thesis against Körner that entailment-statements are contingent statements about expressions (P. F. Strawson, "Necessary Propositions and Entailment-Statements," *Mind*, April 1948). The distinction is carefully observed, on the other hand, by C. Lewy, in the essay "Entailment and Necessary Propositions," in Max Black, ed., *Philosophical Analysis* (Ithaca, N. Y., 1950).

But just in case some nihilist or ultra-empiricist among analytic philosophers should boldly accept the consequence that there are no necessary propositions at all, it is advisable to refute the argument against the *NN* thesis unconditionally. The subtle error of the argument consists in a confusion of *necessity* as a *logical* property of propositions, and *certainty* as a *psychological* state. It is tacitly assumed that "p is necessary" is equivalent to, or at least entails, "p can be known with absolute certainty." Yet, it is easy to see that on this assumption the proposition of arithmetic "$63 \times 45 = 2835$" would be no more necessary than the empirical proposition that day always follows and is followed by night (in fact, for a man with a poor training in arithmetic, the latter proposition would be necessary to a higher degree). The evidence on which one asserts such an equation is that calculations performed by oneself led to this result each time, and that calculations performed by other people competent at multiplication confirmed the result. Nevertheless, it remains logically possible that such a finite series of repetitions of the same calculation should have been infected with errors, and that future calculations should lead to a different result. We may feel confident that no mistake was involved in any of the steps of the calculation, yet it is logically possible that a mistake was made. Remember that the only certainty which Descartes' demon was powerless to undermine was the certainty of "I am uncertain"; the propositions of arithmetic fell a prey to the demon just like the propositions of physics. Some philosophers hold that it is the distinctive mark of empirical propositions that no matter how much confirming evidence may have been accumulated at a given time, it always remains possible that in the future disconfirming evidence will turn up. But this is surely an unsuccessful way of distinguishing empirical propositions from necessary propositions. The more complex a deduction, the greater the probability of a deductive error, and hence the greater the probability

that a future repetition of the deduction should lead to a different result. Yet we know that the proposition which we judge in terms of deductive evidence is either necessarily true or necessarily false.

What marks a proposition as a priori is not that it is capable of being known, either as true or as false, with *absolute certainty*. It is rather that the only kind of cognitive activity which we admit as appropriate to its validation is conceptual analysis [37] and deduction—the "mere operation of thought." In this sense of "a priori," "p is a priori" is itself a priori whether or not it be true, for it is by "the mere operation of thought" that we determine whether a proposition is a priori. For example, if by analysis of truth possibilities I establish the truth of the proposition "if New York is overcrowded, and if it is unpleasant to live in New York if New York is overcrowded, then it is unpleasant to live in New York," without investigating empirically whether the atomic components of this compound proposition are true or false, then I have *by the same analysis* established the *necessary truth* of this proposition. For the analysis establishes that *all* propositions expressed by sentences of the form "if (p and (if p, then q)), then q" are *true*. And if the nonempirical evidence in terms of which p is established as true coincides with the evidence in terms of which p is established as necessarily true, then "p is necessary" is necessary in the same sense in which "p" is necessary. What is decisive is that if a necessary proposition is not mistakenly believed to be contingent, then its truth is known simply as a corollary of its necessity, by the principle "if Np, then p" (whatever is necessarily the case, is the case). To add a simple illustration: by reflecting on the meanings of "square" and "equilateral" we come to know that "x is square" entails "x is equilateral." The entailment, however, can be expressed in the form "it is necessary that there be no square which is not equilateral," and by "if Np, then p" this entails "there is no square which is not equilateral."

What led us into this discussion, ending in final acceptance, of the NN thesis [38] was the doubt whether the definition of "a priori"

[37] "Conceptual analysis" is here used broadly so as to cover also intuitive apprehension of relations between concepts, e.g. that "x is later than y" is incompatible with "y is later than x." In this way the above definition of "necessary truth" allows for intuitively necessary propositions as a subclass of necessary propositions.

[38] It should be noted that the NN thesis is compatible with the view that, where p is a necessary but *nonmodal* proposition. p does not entail Np. "Np entails NNp" must be distinguished from "Np entails that p entails Np." Should one suppose that the latter entailment holds, one would probably do so through fallacious identification of this entailment with the valid entailment "(Np and p) entails Np"; but the principle of exportation which holds for material implication does not hold for entailment. In the essay already cited, Lewy argues against "Np entails that p entails Np," but he does not reject the NN thesis.

in terms of the psychological concept "assent" did not entail that in classifying a proposition as a priori we assert a generalization of psychology. But it now appears that the definition "capable of being known by conceptual analysis and deduction alone," whether or not it be considered "psychologistic," is quite consistent with the view that we are making in no sense an *empirical* statement about a proposition when we classify it as a priori. For, to repeat, the same intellectual operations by which we would normally establish such a proposition as true also establish it as *necessarily* true.

E. Epistemological and Terminological Questions

The main lesson to be learned from the preceding discussion, however, is that the question whether there are *synthetic a priori* truths, which is answered negatively by the logical positivists but which has been resuscitated in recent years as semantic analysis has become more sophisticated, is not a clear question as long as the term "analytic" (and its contrary "synthetic") is used as loosely and ambiguously as it is used in some quarters. I shall, therefore, make some terminological recommendations which I at least, even if nobody else, intend to accept (at least for the remainder of this book). In the first place, it would be desirable not to use "analytic" in the broad sense of "true by virtue of meanings," since this sense is indistinguishable from the sense of "a priori," and the thesis of the analyticity of all a priori truth thus becomes trivial. (Should the reader be tempted to object that an *explication* need not be trivial, and that the thesis "all a priori truth is analytic" may be intended as just that, he is urged to suspend judgment until he has read the next chapter, in which it will be argued that we have to fall back on the "explicandum," the concept of necessary truth, in order to explain the concept of logical truth which enters into the definition of the alleged "explicatum," the concept of strictly analytic truth.) But if, because of some mis-

leading traditional connotations of "a priori" (or "necessary"), philosophers insist on labeling this broad sense "analytic," let them use the more discriminating term *broadly analytic*. There is no reference to logical truths as the ground of analyticity in the definition of "broadly analytic," and hence we need not fear an infinite regress if we classify logical truths (which are characterized by absence of descriptive constants) themselves as broadly analytic, together with such a priori truths as are neither logical truths nor deducible from logical truths with the help of adequate eliminative definitions (i. e. definitions enabling elimination of the defined term). The latter subclass of broadly analytic truths correspond to what are frequently called "statements true by *implicit* definition," which will receive detailed attention in a later chapter. Broadly analytic truths are distinguished from *strictly analytic* truths, which are defined as truths which, though not themselves logical principles, are deducible from logical principles. They are so deducible either *directly*, by substitution of descriptive constants for the bound variables of a logical principle, or indirectly with the help of adequate eliminative definitions. In the former case they might be called *logically true*, in the latter case *definitionally true*.[39] These categories may be illustrated by the following statements: (a) logical principles—for any property f, there is no x such that fx and not-fx; (b) strictly analytic statements which are logically true—nothing is both a cat and not a cat; (c) strictly analytic statements which are definitionally true—all bachelors are

[39] At the end of a recent study ("Bolzano's Definition of Analytic Propositions," *Methodos*, 2, 1950) J. Bar-Hillel recommends a terminology which is substantially "in accord with Carnap's terminology," and I do not wish simply to ignore his proposal, without indicating my reasons for departing from it. Bar-Hillel in effect proposes that "logically true" be used in the sense of "necessarily true (true a priori)," as is evident from his proposal "let us use 'factually true' for 'true but not logically true.'" According to this terminology, however, it would be an open question whether all logically true statements are deducible from logical truths, and for this reason the terminology is misleading. Bar-Hillel further proposes the term "analytically true" for what I would call "trivially analytic": "Let us call sentences of the form '*A-B* is *B*' and '*A-B* is not non-*B*' 'analytically true.'" If we adopt this proposal, we have to say that there are many logical truths which are not analytic, and since "synthetic" is widely used as an abbreviation for "nonanalytic and noncontradictory," we would be saying that many logical truths are synthetic. Bar-Hillel would of course say that "synthetic" should be used in the sense of "factually true or factually false," not in the sense of "nonanalytic and noncontradictory." But this proposal likewise expresses simply the resolution to get rid of the synthetic a priori by terminological fiat.

unmarried; (d) broadly analytic statements which are not strictly analytic—no surface is blue and red at the same time.

It should be kept in mind that in establishing a statement as strictly analytic one presupposes broadly analytic statements, viz. logical truths and—in the case of strictly analytic statements that are not logically true, like "all bachelors are unmarried"—analyses. Now, it is often believed that a philosopher who believes in synthetic a priori knowledge is thereby committed to the postulation of a cognitive faculty, call it the faculty of intuitive apprehension of necessary connections, which an empiricist epistemology must repudiate. An empiricist epistemology holds, I suppose, that the only propositions that we can know to be necessary are those I call "strictly analytic." But there emerges now a gross incongruity: if a synthetic a priori proposition is one that is broadly but not strictly analytic, and broadly analytic propositions constitute the ground of validity of strictly analytic propositions, must not the same faculty be involved in knowledge of strictly analytic propositions as is alleged to be involved in knowledge of synthetic a priori propositions? To be sure, if logical truths could be established by some purely formal procedure, and if similarly analyses of concepts could be formally validated, then the situation would be far different. But I hope to show, gradually, that such a "formalistic" attitude is untenable, and that accordingly we must put up with "intuitive apprehensions of necessary connections" as long as we grant that there are necessary propositions at all. Whether there are necessary propositions that cannot be established as such by recourse to that subclass of necessary propositions called "logical truths" will then appear as a question of far less epistemological interest than has attached to it since the empiricist revolt against the "synthetic a priori."

GLOSSARY

Absolute concepts	contrasted, by Carnap, with *semantical* concepts in that they are not relative to a language (e. g. *L-implication* as a relation between propositions, not sentences)
Analysandum	concept which is (or is to be) analyzed
Analysans	concept expressed by a complex expression and claimed to be identical with analysandum (e. g. brother = male sibling)
Analytic	
broadly	true by virtue of the meanings of constituent terms (*a priori*)
explicitly	substitution instance of a logical truth (*logically true*)
implicitly	not explicitly analytic, but translatable into an explicitly analytic statement with the help of adequate explicative definitions
incompatibility	p and q are analytically incompatible if "if p, then not q" is strictly analytic
strictly	explicitly or implicitly analytic
trivially	statement synonymous with a statement of the form "if p and q, then p," or "all AB are A"
Antecedent	if-clause of a conditional statement
A priori proposition	proposition expressed by a broadly analytic statement; can be known to be true without experience of confirming instances, by just thinking about the proposition
Atomic statement	singular statement which contains no statements as components; ascribes a simple quality to an individual or a simple relation to a set of individuals
Classificatory concept	concept effecting a simple division of things

into those to which it applies and those to which it does not apply (e. g. "hot" in contrast to "warmer" and "degree of temperature")

Codeterminate predicates — predicates designating determinate forms of the same determinable quality, e. g. color predicates. Any two codeterminate predicates are *incompatible*.

Cognitive

- **meaning** — a sentence is said to have cognitive meaning if it is used to express a proposition
- **question** — question capable of being answered by a *sentence that has cognitive meaning*

Comparative concept — concept expressing a comparison, like "harder," "warmer"

Conditional — statement of the form "if *p*, then *q*"

Conjunct — statement conjoined with others

Connective — word used to form a compound statement, like "or," "if-then"

Connotation — a predicate is said to connote those properties which a thing must have in order for the predicate to be applicable to it; contrasted with *denotation*

Consequent — *then-clause of a conditional*

Contingent proposition — *p* is contingent if *p* as well as not-*p* is logically possible

Contradictory — *the contradictory of p is that proposition* which must be false if *p* is true and true if *p* is false

Converse relation — see *relation*

Coordinate language — language in which qualities and values of magnitudes are ascribed to space-time positions or regions

Counterfactual conditional — conditional which asserts what *would* be the case if a condition, which in fact is not realized, *were* realized

Decision procedure — procedure for deciding a logical question, e. g. whether a sentence is a consequence of, or contradicts the axioms of, a system

Deductively valid — an argument is deductively valid if the conclusion follows with logical necessity *from the premises*; in other words, if it is a contradiction to assert the premises and at the same *time* to deny the conclusion

Definiendum — expression which is defined

Definiens — complex expression by which the definiendum is defined

Definientia — plural of *definiens*

Definition

- **circular** — the *definiendum* occurs in the definiens, or a part of the definiens is defined in terms of the definiendum
- **coordinative** — as used by Reichenbach, interpretation of the terms of a formal deductive system by means of expressions denoting observable objects or processes (e. g. definition of "length" in terms of a standard rod)
- **eliminative** — enables elimination of definiendum from any sentence in which *it occurs* ("to define a term is to show how one can get along without it")
- **explicative** — analysis of the meaning of the definiendum
- **explicit** — definiendum can simply be replaced by definiens in any sentence without changing the remainder of the sentence (e. g. father = male parent)
- **implicit** — a set of postulates (axioms) is said to implicitly define the primitive terms in it, i. e. it delimits their denotations to objects and relations that satisfy it
- **in use (contextual)** — rule for translating sentences containing definiendum into synonymous sentences that do not contain *it*; but while being eliminative, it is not explicit (e. g. *x* is brother of *y* = *x* is male and has the same parents as *y*)

ostensive	explaining the meaning of a term by pointing at, or inducing experience of, instances denoted by it
recursive	rule for eliminating definiendum in a finite number of symbolic transformations from expressions in which it occurs together with constant arguments (e. g. "+" from "3 + 2")
Denotata	objects of which a term is truly predicable (e. g. individual men are denotata of "man")
Denotation	class of all the denotata of a term; also called *extension* of the term; also used in the sense of *denotatum*
Designator (designative expression)	expression referring to objects, events, relations, properties, etc.; contrasted with "syncategorematic" words, like "of," "or," "the," also with purely emotive words
Designatum	Carnap: that which is designated by a designator; C. W. Morris: connotation
Descriptive	
predicate	word designating a sensory quality or relation, or a characteristic whose presence can be inferred from what is observed
semantics	empirical study of the meanings of expressions of natural languages
term (constant)	descriptive predicate, or name of a concrete entity, or description in terms of descriptive predicates of a concrete entity
Dichotomous concept	classificatory concept
Dispositional property	tendency to react in a certain way to a certain kind of stimulus (in a generalized sense of "stimulus"); e. g. "soluble," "fragile," "irritable"
Domain (converse domain) of a relation	see *relation*
Empty predicate (concept)	predicate of which there are no denotata, e. g. "unicorn," "man who is 30 ft. tall"

Glossary

Equivalence	
extensional	two predicates are extensionally equivalent if they have the same extension though they may differ in connotation
logical (strict)	see *logical*
material	two statements are materially equivalent if they materially imply each other
Essential occurrence	nonvacuous occurrence
Existential	
generalization	the inference of "something has property P" from "x has P" where x is a particular entity (e. g. cruelty is hated, therefore something is hated)
quantifier	the expression "there is an x"—or its idiomatic equivalents "something," "somebody," etc.
statement	statement asserting that something (one or more) has a specified property, without saying which it is (they are)
Explicandum	concept to be explicated (analyzed) in terms of a more precise concept
Explication	process of analyzing (explicating) a concept already used in everyday or scientific discourse, sometimes within the framework of a "language system"
Explicating (or analytic) synonym	definiens of an explicative definition
Explicatum	correlative to "explicandum" the way "analysans" is correlative to "analysandum"
Explicit contradiction	a conjunction of statements is an explicit contradiction if at least two conjuncts are contradictories of each other (e. g. "p and q and not-p," but not "p and (if p, then q) and not-q")
Explicitly analytic	see *analytic*
Extension	see *denotation*

Extensional	
connective	connective used to form compound statements which are truth-functions of the component statements (e. g. "and," but not "if-then" in most ordinary uses)
context	a sentence is an extensional context for a constituent predicate, if replacement of the latter with a predicate of equal extension does not change its truth-value; it is an extensional context for a constituent sentence, if it is a truth-function of the latter; and for a constituent name (or description) if replacement of the latter by one that denotes the same object does not affect the truth-value of the sentence
language	language whose compound statements, i. e. statements containing parts which are themselves statements, are truth-functions and whose noncompound statements have a truth-value which depends only on the extensions of the predicates they contain (excludes, e. g., sentences of the forms "it is necessary that p," "A believes that p")
sentence	sentence which is an extensional context for all constituent expressions
Extralinguistic	outside of language
Extralogical axioms (postulates)	axioms which are not logical truths in their intended interpretation, e. g. the axioms of a system of geometry, or of mechanics
Factual	
content	a statement is said to have factual content, by logical empiricists, if it is neither self-contradictory nor analytic (it "says something about the world")
implication	see *implication*
truth	true contingent proposition
Factually empty	devoid of factual content (not to be confused with Carnap's term "F-empty": a predicate which is empty though it is logically possible that it should denote)

Formal	
contradiction	negation of a logical truth, i. e. false by sole virtue of the meanings of logical constants
entailment	p formally entails q if "if p, then q" is a logical truth
deduction	deduction without attention to the meanings of nonlogical constants
implication	see *implication*
mode of speech	speaking about words in discussing a philosophical problem; contrasted with *material* mode of speech
postulate	postulate whose nonlogical constants are given no particular interpretation in making deductions from it
Formation rules	rules specifying what sequences of what kinds of symbols are sentences (formulae)
Functional logic (calculus)	logic as including not only the propositional calculus but also the theory of quantification (see *quantifier*); includes syllogism theory as well as the theory of relations
Functor	expression designating a magnitude, mathematical or physical, e. g. "sum," "length"
Higher functional calculus	see *lower functional calculus*
Implication	
factual	implication by virtue of an empirical law, e. g. "if a block of ice is exposed to 90° F, then it melts"
formal	a propositional function Fx formally implies a propositional function Gx if the universal statement "for every x, if Fx then Gx" is true and extensional
logical (strict)	implication which is logically true
material	p materially implies q if either p is false or q is true—though p and q may be wholly unrelated in meaning
probability	a probability implication from p to q

	obtains if q has a certain probability relative to p (a formal implication is an extensional probability implication of the highest degree)
Implicit contradiction	conjunction from which an explicit contradiction is deducible though it is not itself an explicit contradiction
Indicator term	term whose reference constantly changes with the context in which it is used, like "this," "now," "here," "I"
Individual	
constant	name of a particular
variable	variable whose substituends are individual constants
Intension	of a predicate: connotation; of a sentence: proposition expressed by it
Intensional vagueness	a predicate or common noun is intensionally vague if the set of connoted properties is not fixed (e. g. would a human being with female reproductive organs but otherwise masculine body be a man or a woman?)
Interpreted language	language whose expressions are not only connected by syntactic rules but also have semantic meaning (reference to something outside of language)
L-determinate	*L*-true or self-contradictory
L-equivalent	logically equivalent (see *logical*)
L-true	logically true (see *logical*)
Language system	language defined by an explicit listing of *primitive (undefined) vocabulary*, both logical and descriptive, and by the following kinds of rules: rules of sentence formation, rules of deduction (transformation), semantic rules
Level of a predicate	predicates connoting qualities or relations of individuals belong to the first level, predicates connoting properties or relations

	of such qualities or relations belong to the second level, and so on (e. g. "is a color," "is a desirable quality" are second-level predicates)
Logical	
construction	an entity A is said to be a logical construction out of a specified set of entities S, if the expression "A" which denotes A is contextually definable by reference to members of S (in this sense physical objects are, according to phenomenalism, logical constructions out of sense data)
equivalence	p and q are logically equivalent if they logically imply each other
independence	two propositions are logically independent if they are compatible and none logically implies the other; in a derivative sense, predicates are said to be *l. i.*
possibility	state of affairs which is conceivable without self-contradiction
truth (principle)	necessary proposition which contains only logical concepts (for the meaning of "logical concept—or constant," see Chap. 6)
type	two expressions a and b are of the same logical type if for any sentential function "Fx," "Fa" and "Fb" are either both meaningful or both meaningless sentences; the logical type of an expression is the class of expressions of the same logical type as itself. In a parallel sense, one speaks of the logical type of an *entity*
Logically	
proper name	name which is meaningless unless it denotes something; contrasted with proper names which are abbreviations for descriptions which may or may not denote something, e. g. "Apollo"
true	a necessary statement which contains only logical constants essentially; in a narrower sense, substitution instance of a logical

	truth, but not itself a logical truth (principle)
Logicist	the Frege-Russell philosophy of mathematics according to which mathematics is a branch of logic
Logistic language	language using the symbolism and syntax of symbolic logic
Lower functional calculus	that part of functional logic in which only *individual* variables are employed; contrasted with the *higher* functional calculus in which also variables ranging over abstract entities, like properties and classes, are used
Material	
criterion of adequacy	implicitly analytic statement containing a term whose meaning is to be explicated in such a way that the statement becomes explicitly analytic
entailment	entailment expressed by a conditional which is not logically true
mode of speech	contrasted with *formal* mode of speech: discussing a philosophical problem in the object language, talking about extralinguistic entities
Meaning postulate	postulate of a language system which is broadly but not strictly analytic; serves to delimit possible interpretations of descriptive primitive terms
Mention of an expression	contrasted with *use* of an expression, i. e. speaking about the expression itself, not about its denotata (e. g. " 'men' is the plural of a noun " vs. " men cannot live without women ")
Metalanguage	language used to talk about language
Metalogical	discourse about logic (e. g. the statement that the propositional calculus as axiomatized in *Principia Mathematica* is complete, is metalogical)
Mentalistic term	term designating a subjective, not publicly

	observable, state of consciousness; contrasted with *behavioristic*
Metrical concept	concept of a magnitude, like velocity, temperature, degree of blood pressure
Modal	
logic	logic using modalities, i. e. concepts like " possible," " necessary," and therefore containing nonextensional statements
statement	nonextensional statement using modalities
Modalities	see *modal logic*
Model of a set of postulates	an ordered set of entities which satisfies all the postulates, in other words, of which all the postulates are true (" entities " in the broadest sense, including relations)
Modus ponens (*ponendo ponens*)	the principle that whatever proposition is implied by true propositions is itself true
Modus tollens (*tollendo tollens*)	the principle that whatever proposition implies a false proposition is itself false
Molecular statement	singular statement composed of atomic statements
Natural language	contrasted with *language system*; the rules of a natural language are implicit in the use of the expressions, but most of them are not explicitly formulated; further, a natural language is characterized by ambiguity and vagueness
Necessary proposition (truth)	proposition which cannot possibly be false
Nominalistic language	contains in its primitive vocabulary no names except names of individuals, and no variables except individual variables (such languages are extensional, but an extensional language need not be nominalistic)
Nomological implication	implication expressing an intensional connection, logical or causal, unlike material and formal implications, which are extensional sentences

Non-natural (property, entity)	not given in sense experience, or capable of being so given; e. g. propositions, as distinguished from sentences, are usually held to be thinkable but not sensible
Object language	contrasted with *metalanguage*: language used to talk about extralinguistic objects. In a relative sense, however, a metalanguage may itself be object language relative to a meta-metalanguage which is used to talk about it
Occurrent concept	concept referring to occurrences (events); usually contrasted with *dispositional concept*
One-place predicate	nonrelational predicate
Pragmatic	
contradiction	sense (1): proposition which must be false if it is asserted, but could be true if it were not asserted; sense (2): proposition whose falsehood may be inferred with high probability from the fact that it is asserted
meaning	states of the sign user or sign interpreter which are causally connected with the occurrence of the sign (e. g. "it will rain" pragmatically means that the speaker believes that it will rain, also that he does not believe that it rains at the time of utterance of the sentence)
Pragmatics	the science of pragmatic meaning
Pre-analytic	before analysis; e. g. pre-analytic understanding of a predicate = understanding of the predicate before having analyzed its meaning
Predication	statement ascribing a predicate to something, e. g. "he is a general" is a predication of "is a general"
Primitive predicate	predicate which is not defined, except ostensively
Principle of deduction	statement as to which propositions are deducible from which, referring only to the forms of propositions (e. g. *modus ponens*)

Proposition	anything which is not a sentence and can significantly be said to be true or false; state of affairs which may or may not be actual (if it is actual, it is a fact, or a true proposition); anything which can be the meaning of a declarative sentence; anything which may be believed or disbelieved
Propositional	
function	sense (1): expression containing one or more free variables such that a meaningful sentence (true or false) results when suitable constants are substituted for the variables; sense (2): that which is expressed (designated) by a propositional function in sense (1). In this sense a propositional function seems undistinguishable from a property, in the broad sense including relations
logic (calculus)	that part of logic in which nonmolecular statements are treated as units, and are not further analyzed (in traditional terminology, only relations between *propositions*, not between *terms*, are dealt with)
variable	variable whose range consists of propositions
Psychologism	the tendency to confuse *logical* issues with *psychological* issues; e. g. if one tried to answer a question of logical validity by investigating actual beliefs (however, the meaning of this deprecatory word is unclear to the extent that the meaning of "logical" is unclear)
Psychophysical law	statement of a correlation between a mental and a bodily (or behavioral) state
Qualitative predicate	predicate designating a quality, and not defined in terms of names of particulars (e. g. "solar," "higher than Mt. Everest" are not qualitative predicates)

Quantifier	expressions like "some," "all," "for every," "there is," by means of which general statements are constructed
Range	
of a proposition (sentence)	class of state descriptions (states) in which the proposition (sentence) is true; the greater the range of a proposition, the more possibilities are left open by it
of significance	the range of significance of a predicate "P" is the class of values of "x" for which "Px" is true or false (e. g. numbers are outside the range of significance of "blue")
of a variable	class of entities whose names are substitutable for the variable
Realistic language	as contrasted with *nominalistic* lauguage, it contains variables ranging over abstract entities, and possibly also names of such
Relation	
asymmetrical	for all x and y, if xRy, then not-yRx
converse	the converse of R is the relation $\breve{R}$ such that "xRy" is equivalent to "$y\breve{R}x$"
domain	class of objects baving a given relation to something
converse domain	class of objects having the converse relation to something
field	sum of domain aud converse domain
intransitive	for all x, y, z, if xRy and yRz, then not-xRz
irreflexive	relation which nothing has to itself
many-one	if (xRy and uRz) implies $y = z$
many-many	neither many-one nor one-many
one-many	if (xRy and uRz) implies $x = u$
one-one	one-many and many-one
reflexive	R such that if x has R to something, then xRx (sometimes distinguished from *totally reflexive*: R which everything has to itself)
symmetrical	for all x and y, if xRy, then yRx
transitive	for all x, y, z, if xRy and yRz, then xRz
Relational predicate	predicate designating a relation

Reduction	
basis	set of primitive predicates on the basis of which other predicates are introduced into a language either by definitions or by reduction sentences
sentence	sentence which is not an eliminative definition but describes a kind of test for deciding empirically whether a given property which is not directly observable is present
Self-consistent	not self-contradictory
Self-contradictory	proposition from which a contradiction is deducible without presupposing any contingent propositions
Semantic	
concept	concept referring to semantic meanings of expressions (e. g. truth as ascribed to sentences)
meaning	of a sentence: the state of affairs which must exist if the seutence is to be true; of a predicate: connotation, intension
rule	specification of the semantic meaning or the denotation or the denotatum of a designative expression (definitions are semantic rules if the terms constituting the definientia are already understood)
Semiotics	theory of signs (Morris)
Sense	Frege's term for connotation, intension; but Frege applies it also to names and definite descriptions, e. g. if "Shakespeare is the author of *Hamlet*" is an informative statement, then "Shakespeare" and "the author of Hamlet" differ in sense
Sense-data statement	statement about sensations, not about physical events or objects
Sentential	
function	*propositional function* in sense (1)
variable	variable whose values are sentences (and whose substituends are names of sentences)
Singular statement	statement containing no quantifiers

State description	as a term of semantics, complete description, in the form of a conjunction of atomic statements or of atomic statements and negations of atomic statements, of a possible world
Substituend	term which is substitutable for a variable, name of a value of a variable
Substitution instance	statement derivable from a universal statement by substituting the same constant for each occurrence of a variable bound by the same universal quantifier ("for every x," "(x)"); also statement derived by substitution from a propositional function
Syntactic rule	rule governing manipulation of symbols without regard to semantic meaning
Synthetic	not (strictly) analytic but self-consistent
Tautology	sense (1): compound statement which is true regardless of the truth-values of its components; sense (2): propositional function all of whose instances are tautologies in sense (1); sense (3): logically true statement
Theoretical concept	concept referring to something which is postulated in order to explain the observed, but is not directly observable; e. g. electron, gravitational potential, unconscious wish
Thing language	the prescientific language of everyday life in which we speak of things and their observable qualities; contrasted with both sense-data language and theoretical languages of quantitative sciences
Time independent	a property P or relation R such that it is meaningless to say "x has P at time t" or "x has R to y at time t"; e. g. equality as a relation between numbers, entailment as a relation between propositions
Transformation rule	principle of deduction; in a wider sense, also definitions conceived as rules of substitution of symbols

Truth	
condition	semantic meaning of a sentence
function	propositional (sentential) function constructed by means of connectives such that the truth-values of its instances are uniquely determined by the truth-values of the component propositions (sentences); instances of truth-functions (to which "truth-function" is sometimes also applied), therefore, are extensional with respect to the component sentences
table	table constructed in order to define an extensional connective in terms of possible combinations of truth-values of the connected statements, or in order to decide on the basis of such tables whether a given truth-function is tautologous, self-contradictory, or neither
value	truth or falsehood
Two-valued logic	logic assuming that every proposition is either true or false
Type vs. token	repeatable pattern of a linguistic expression, as distinguished from different instances of the same pattern (e. g. an ambiguous word is a type such that not all tokens of the type are synonymous)
Use of an expression	see *mention*
Vacuous occurrence	a term occurs vacuously in a sentence S if the truth-value of S remains unchanged under replacements of that term by any other grammatically admissible term; a term occurs vacuously in an argument if the validity or invalidity of the argument does not depend on its meaning
Value of a variable	member of the range of the variable
Variable	
bound	variable occurring in a sentential function to which a quantifier containing it is prefixed
free	variable which is not bound